Society of Actuaries

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# Calibration of a Regime-Switching Interest Rate Model

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# Context for the Model

- Long-Rate Anchor: 20 Yr, Not (yet) Whole Curve
- Stress-testing
  - Not Forecasting
  - Not Pricing
- What's Important:
  - Severe but Plausible Extreme Scenarios
    - Plausible: in historical context
    - Severe: represent real stresses
    - Extreme: on both (all) tails
- Much Less Important:
  - Accuracy Around the Likely Scenarios
- Completely Irrelevant:
  - Risk Neutrality
  - Arbitrage Free

# Summary

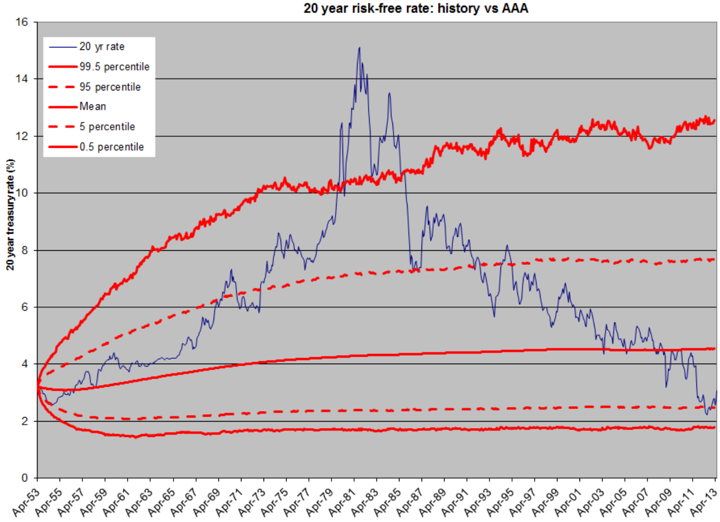
- Typical Generators (e.g. AAA).....
  - Gaussian-based volatility driver
  - A single mean reversion point (MRP)
- .....Fail To Produce Historically Plausible Ranges of Results
  - Unhistorical shape to the realized volatility
  - Tightly bunched paths versus historical ranges
  - MRP assumption largely drives the extreme paths
- To Fix the Problems
  - Use fat-tailed volatility driver
  - Randomize MRP to spread range of extreme paths
- But This Introduces More Parameters
  - Calibration becomes a real challenge

# History of 20 Year US Treasury Rate

Plausible By Definition

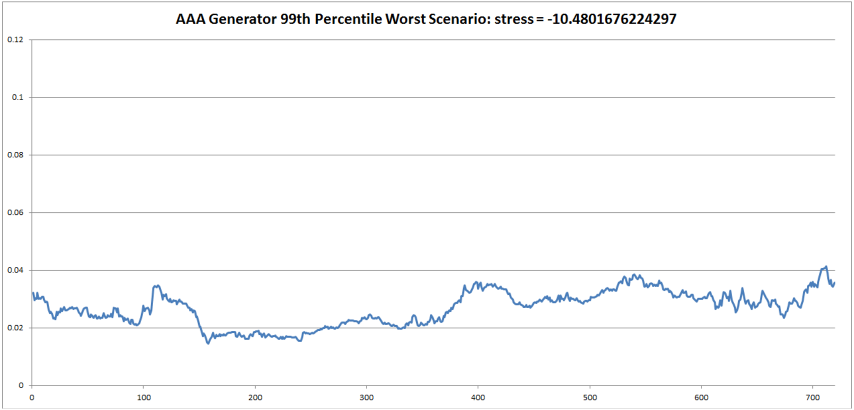
# 20 Yr Treasuries: History vs AAA Generator Monthly %-iles

Neither Early 80's Nor Japan Are Remotely Plausible In AAA

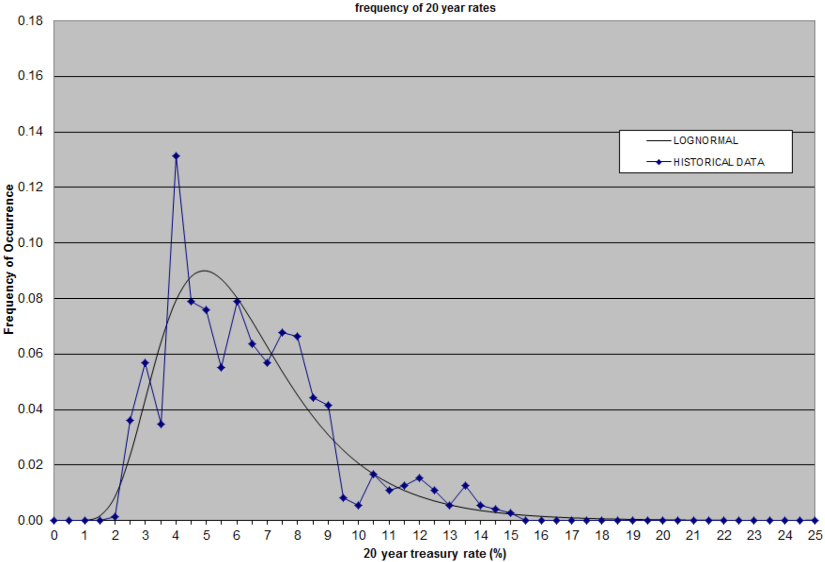


# No One Path Follows the Monthly Extremes

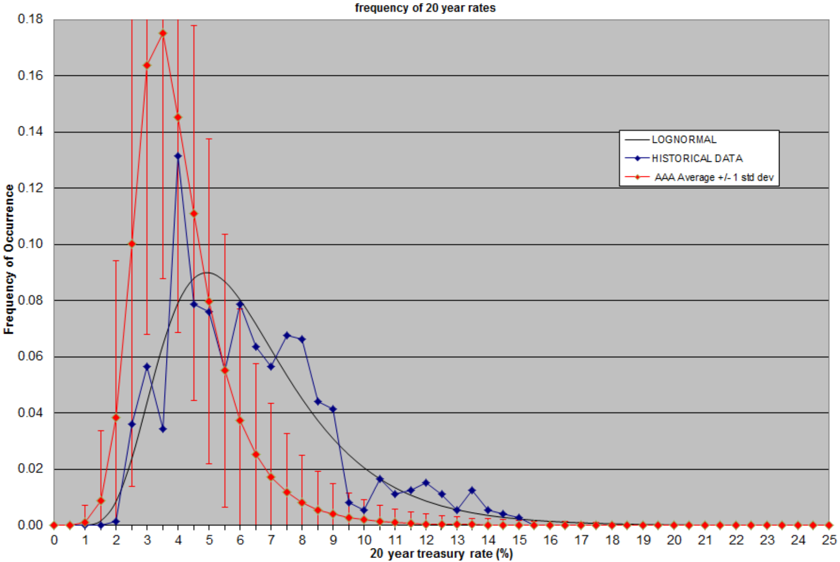
AAA Extreme Paths Are Not Japan-Like Near-Term - But They Persist



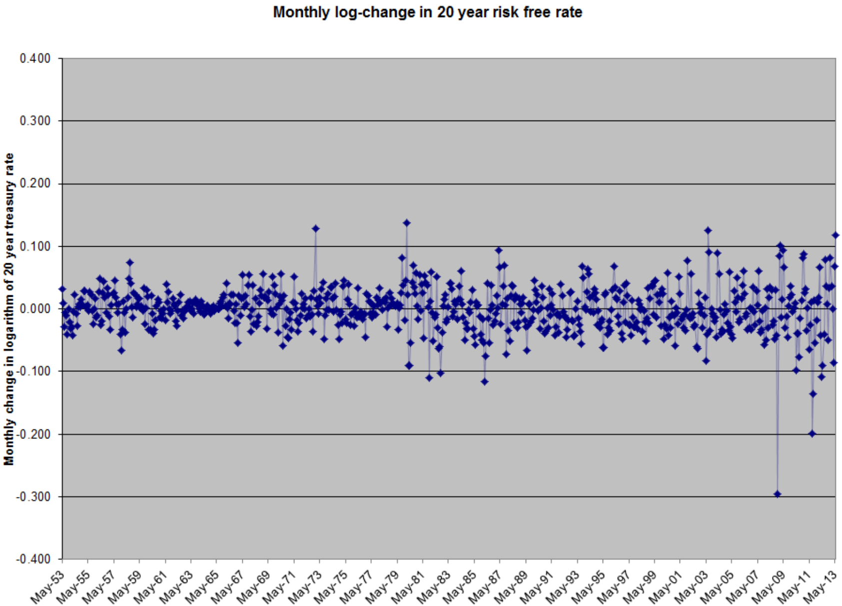
# Historical Frequency of 20 Year Rates



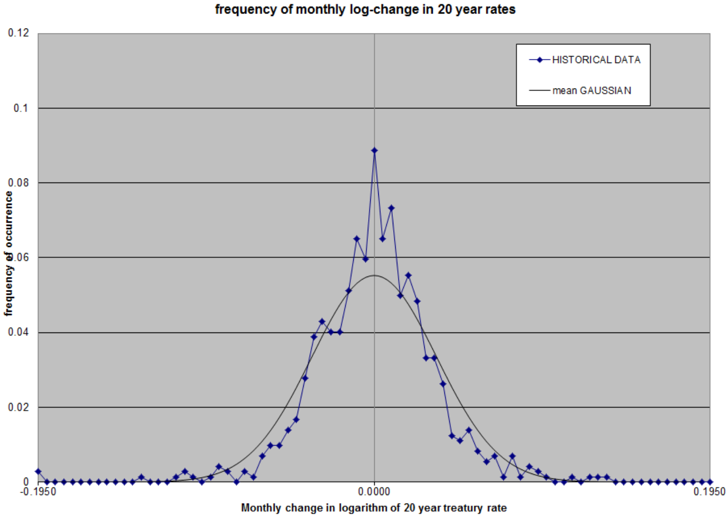
# Historical Frequency of 20 Year Rates vs AAA Generator



# Historical Realized Volatility of 20 Year Rates

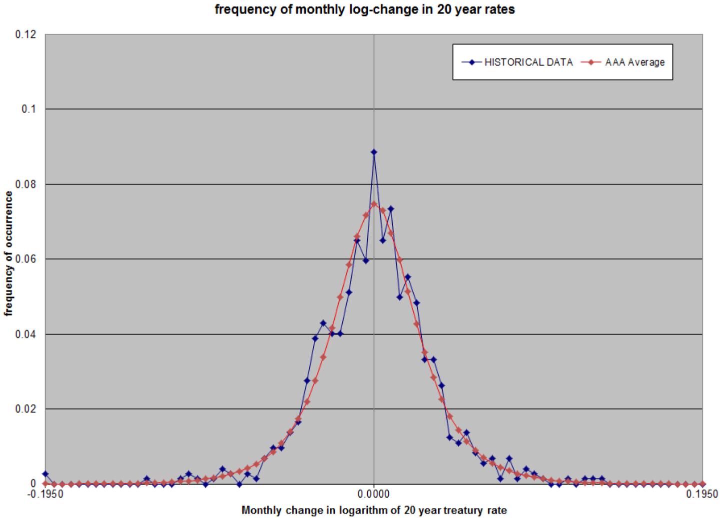


## High Kurtosis

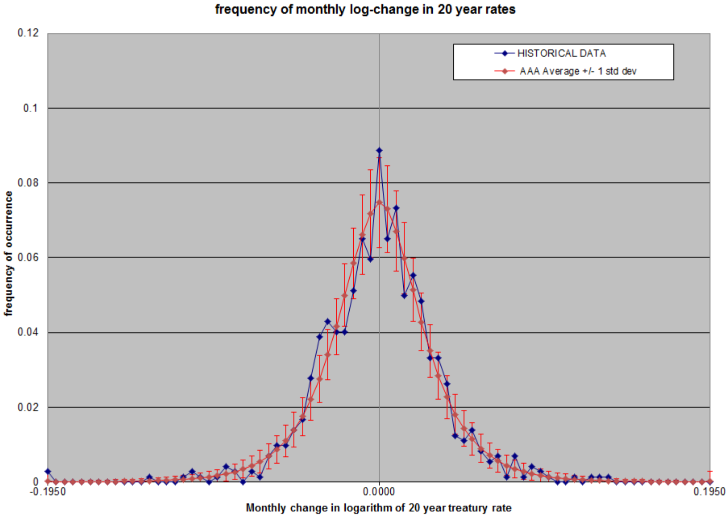


# Historical Distr. of Realized Volatility vs AAA Generator

Stochastic Volatility Helps, May Not Fully Pick Up The Tails



## Missing Tails Are Significant



## Rate Levels and Spread as well as the Shape of the Realized Volatility Differ Significantly from History

| | **60 Year History** | **AAA Mean** | **AAA StdDev** |
|---|---|---|---|
| **Rate = 20 Year Treasury** | | | |
| Rate Mean | .0635 | **.0410** | .0081 |
| Rate StdDev | .0266 | **.0117** | .0058 |
| Rate Kurtosis (normal=3) | 3.53 | 3.02 | 1.29 |
| Rate 6th-osis (normal=15) | 21.5 | 17.7 | 26.1 |
| (6th Ctrl Mom/StdDev^6) | | | |
| **Realized Volatility = $\Delta \ln$Rate** | | | |
| Volatility StdDev | .0360 | .0338 | .0039 |
| Volatility Kurtosis (normal=3) | 10.9 | **5.3** | 1.6 |
| Volatility 6th-osis (normal=15) | 479 | **76** | 124 |

# Consider A New Model

- Traditional Models (including AAA)

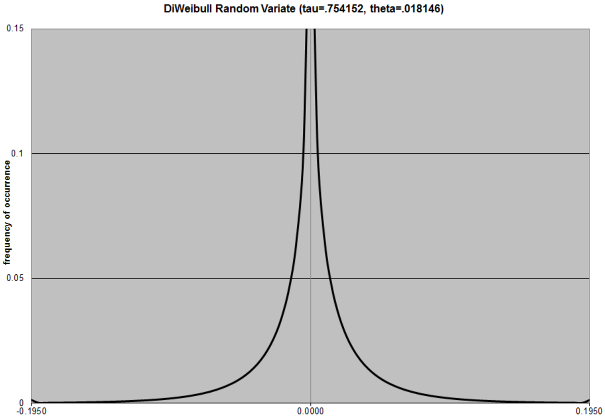
$\Delta \ln Rate_t =$
$F * (\ln MRP - \ln Rate_{t-1}) + SlopeAdjustment + (1 - F) * Gaussian\Delta$

- Proposed New Model:

Regime-Switching with Random Regimes

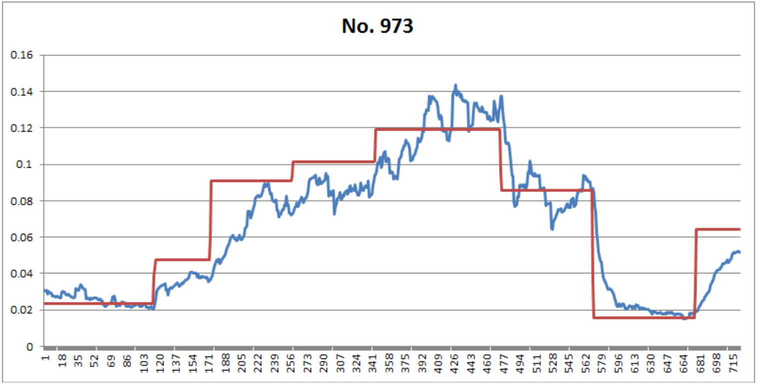
$\Delta \ln Rate_i =$
$F * (\ln MRP_t - \ln Rate_{t-1}) - DriftCompensation + (1 - F) * DiWeibull\Delta$
where
$MRP_t = MRP_{t-1}$
unless
$t - t_{regime} >$a random $Gamma(\alpha, \beta)$ variate.
In that case, the regime switches to a new, random *MRP*:
$MRP_t =$a random *LogNormal* variate, fixed until next regime-switch.
And the regime-switching clock restarts at $t_{regime} = t$.
(a *SlopeAdjustment* can be included if desirable)

**DiWeibull Is Like Laplace:**
**Laplace is symmetric Exponential, DiWeibull is symmetric Weibull**




**Very Heavy Tail**

# A Sample Path From the New Model (inti-MRP 4-53)



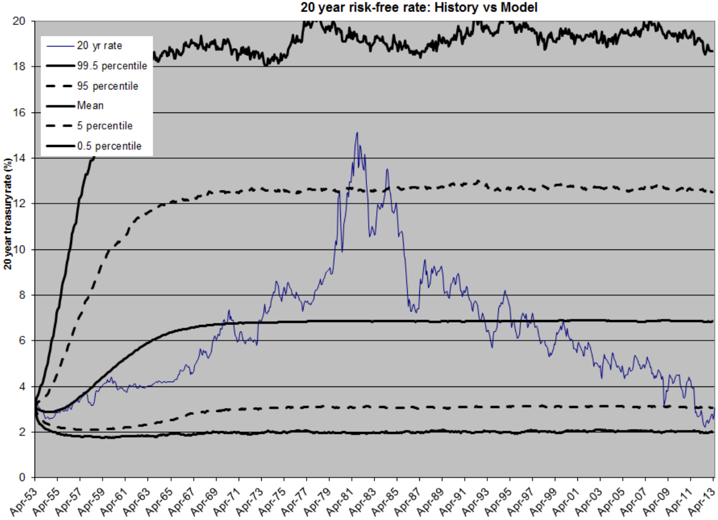
# New Model Requires 8 Parameters

- 2 Parameters For The Regime Clock Random $Gamma(\alpha, \beta)$ Variate.
  - $\alpha = 7.1$ and $\beta = 1.14$ (in annualized units) follows from MLE applied to historical random MRP estimates derived by Least Square Error analysis versus historical rates
  - Average length of an interest rate regime is $\alpha\beta = 8$ Years plus 1 Month
- 1 Initial Value For The MRP
  - Least Square Error analysis versus historical rates gives
    - For 4-1953 start: init-MRP=2.36%
    - For 6-2013 start: init-MRP=2.04%
- This Leaves 5 Parameters To Be Determined
  - 2 Parameters For The Lognormal Random MRP
  - 2 Parameters For The *DiWeibull*$\Delta$ Volatility Driver
  - 1 Mean Reversion Strength Factor ($F$ in the formula)
- Choose The 5 Parameters To Best Align Comparative Statistics vs History

## Rate Levels and Spread as well as the Shape of the Realized Volatility Now Align With History

| | 60 Year History | Model Mean | Model StdDev |
|---|---|---|---|
| **Rate = 20 Year Treasury** | | | |
| Rate Mean | .0635 | **.0631** | .0126 |
| Rate StdDev | .0266 | **.0268** | .0105 |
| Rate Kurtosis (normal=3) | 3.53 | 2.96 | 1.24 |
| Rate 6th-osis (normal=15) | 21.5 | 15.8 | 18.9 |
| (6th Ctrl Mom/StdDev^6) | | | |
| **Realized Volatility = $\Delta$ lnRate** | | | |
| Volatility StdDev | .0360 | .0363 | .0027 |
| Volatility Kurtosis (normal=3) | 10.9 | **10.9** | 4.8 |
| Volatility 6th-osis (normal=15) | 479 | **365** | 636 |

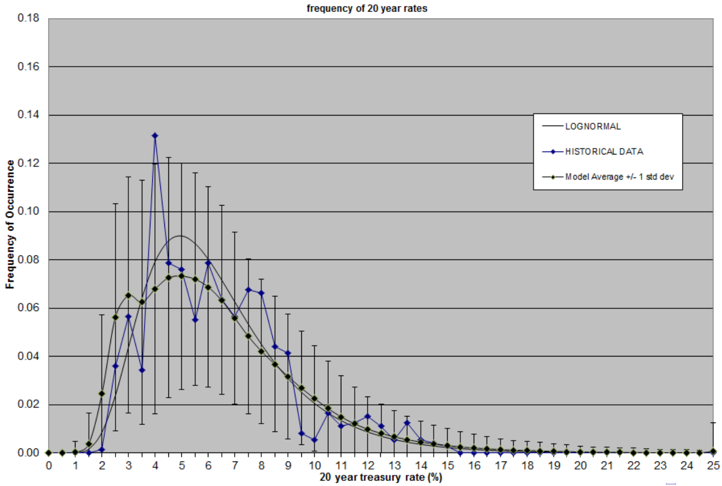
# New Model (init-MRP 4-53) vs History: Monthly %-iles




Only 55/723 Months Breach 5%-95%: History Fits Into This Easily

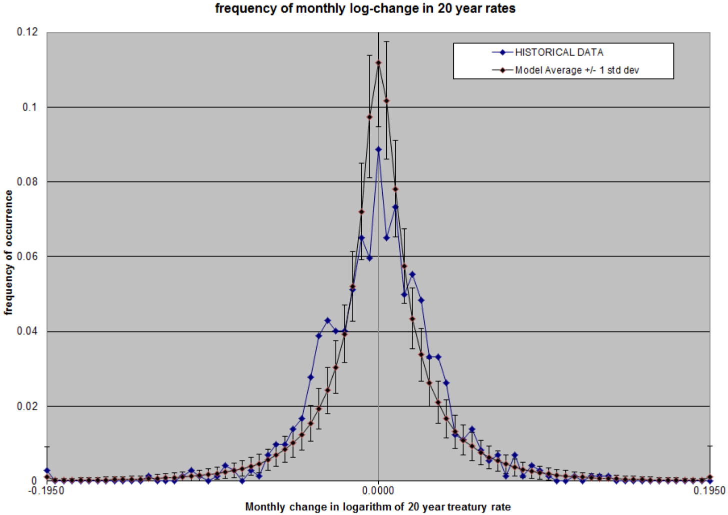
# Hist Freq of 20 Yr Rates vs New Model (init-MRP 4-53)

Fits Like A Glove

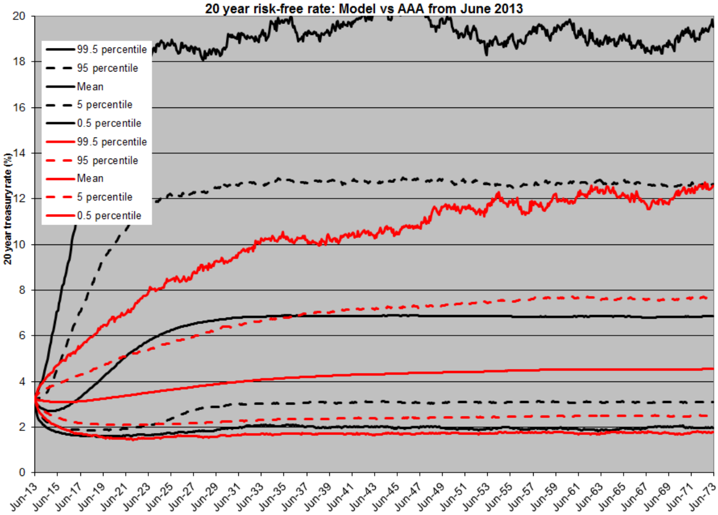


# Realized Vol: History vs New Model (init-MRP 4-53)

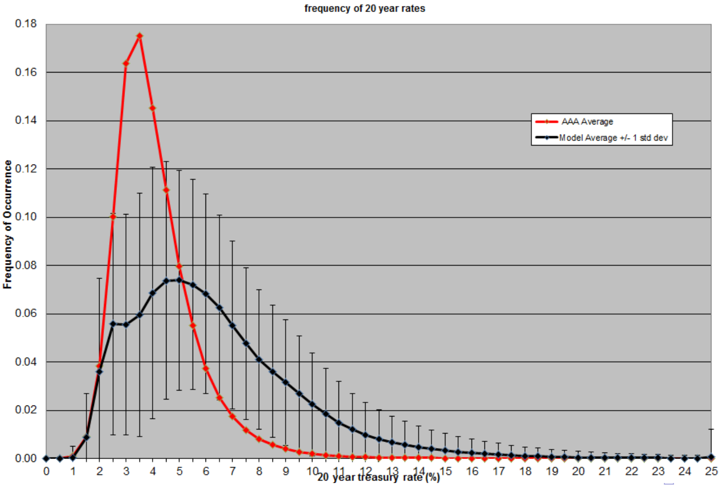
## Too Far In The Other Direction? At Least The Tail Is Good



# AAA Vs New Model (init-MRP 6-13): Monthly %-iles

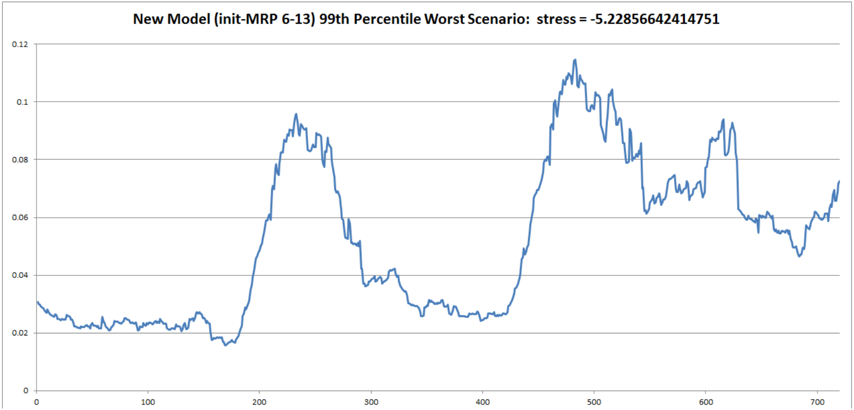


## Same Prob. $\leq$ 2.25%, Wild Difference Thereafter



# An Extreme Path In The New Model (init-MRP 6-13)

For First 15 Years Slightly More Stress Than The 99%-ile AAA Scenario (And After 15 It Has Different Stresses That AAA Would Never Generate)

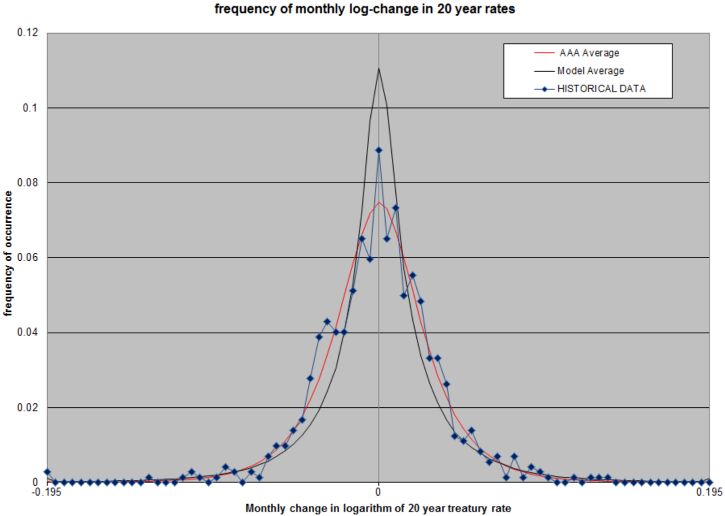


## Shape Of Model Realized Volatility Is Not Only Fatter-Tailed On Average But Also Much More Varied

| | Model Mean | Model StdDev | AAA Mean | AAA StdDev |
|---|---|---|---|---|
| **Rate = 20 Year Treasury** | | | | |
| Rate Mean | .0628 | .0126 | .0410 | .0081 |
| Rate StdDev | .0271 | .0104 | .0117 | .0058 |
| Rate Kurtosis (normal=3) | 2.94 | 1.19 | 3.02 | 1.29 |
| Rate 6th-osis (normal=15) | 15.3 | 17.7 | 17.7 | 26.1 |
| (6th Ctrl Mom/StdDev^6) | | | | |
| **Realized Volatility = $\Delta$ lnRate** | | | | |
| Volatility StdDev | .0364 | .0027 | .0338 | .0039 |
| Volatility Kurtosis (normal=3) | 10.8 | **5.0** | 5.3 | **1.6** |
| Volatility 6th-osis (normal=15) | 368 | **706** | 76 | **124** |

# Realized Vol: New Model (init-MRP 6-13) vs AAA

Both Miss Parts of Historical Volatility Shape Despite Other Evidence



# Calibrate Instead On Direct Shape Statistics

Instead of Kurtotis and 6th-osis:

- Minimize L2 Distance of CDF to History

$$\sqrt{\int (F(r) - H(r))^2 \, dr}$$

- Minimize L1 Distance of CDF to History

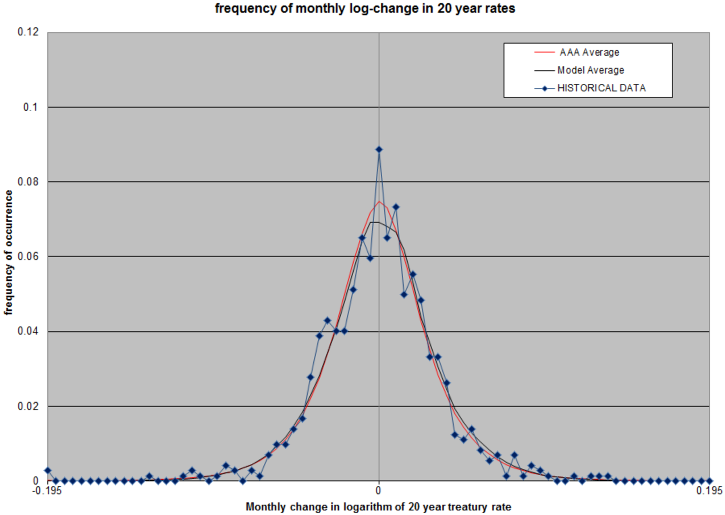
$$\int |F(r) - H(r)| \, dr$$

- Use CDF Rather Than PDF To Emphasize Tails
- Use Both Rates and Realized Volatility

# Calibration On L2 and L1 Distance, Means, Vol Std Dev

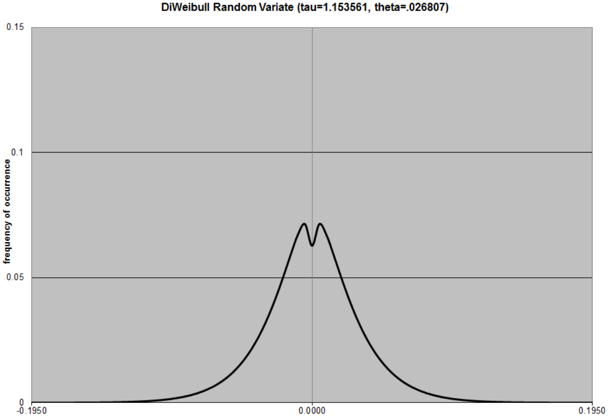
| | Model Mean | Model StdDev | AAA Mean | AAA StdDev |
|---|---|---|---|---|
| **Rate = 20 Year Treasury** | | | | |
| Rate Mean | .0631 | .0078 | .0410 | .0081 |
| Rate StdDev | .0190 | .0048 | .0117 | .0058 |
| L2 Distance to History | .0372 | .0135 | .0858 | .0271 |
| L1 Distance to History | .0102 | .0035 | .0230 | .0070 |
| **Realized Volatility = $\Delta \ln$Rate** | | | | |
| Volatility StdDev | .0335 | .0018 | .0338 | .0039 |
| L2 Distance to History | .0067 | .0012 | .0074 | .0030 |
| L1 Distance to History | .0027 | .0004 | .0031 | .0013 |

# Realized Vol. Comparison For This Alternative Calibration

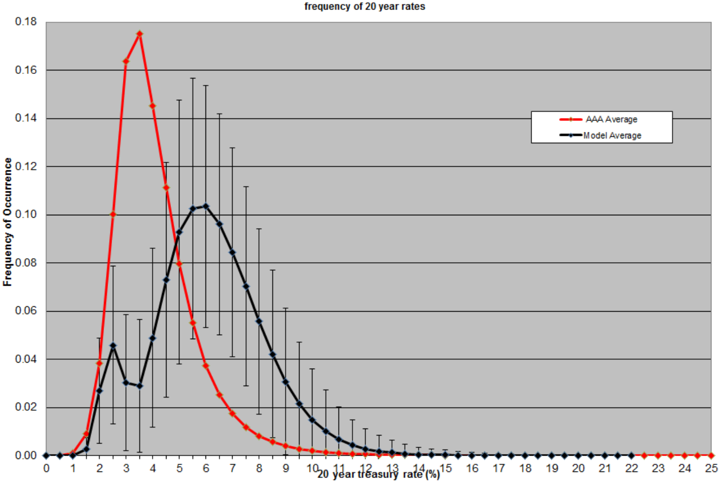


# DiWeibull Driver For This Alternative Calibration

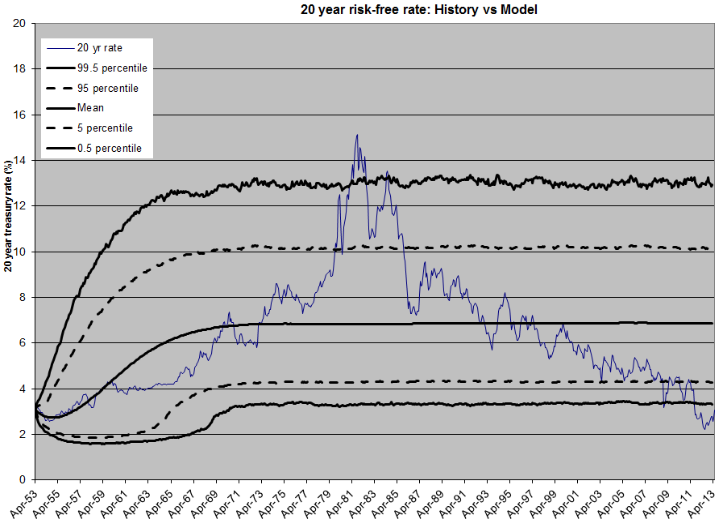
With This Calibration The Volatility Driver Has Milder Tail
BiModal Not A Problem: Mean-Reversion Smooths It Out In Realized Vol.



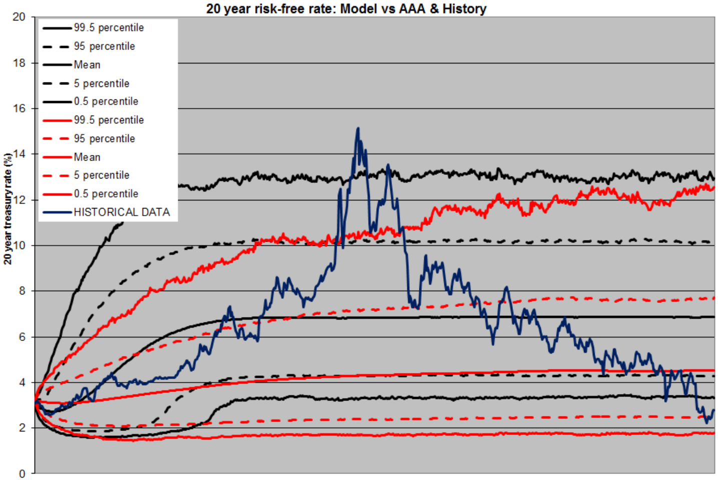
# Rate Distr. Comparison For This Alternative Calibration



# Monthly %-iles vs History For This Alternative Calibration



# And Compared To AAA Generator



## Still Japan-like For A Good 15 Years