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Actuarial Utility and Preference Functions

April 2019

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CONTENTS

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“Be not the first by whom the new is tried, nor the last to put the old aside.”

—Alexander Pope

Abstract

Utility functions were developed on reasoned principles and are used in economic models to determine optimal economic results based on normative behaviors and the assumption that people will choose the most economically advantageous option presented to them. This approach does not always accurately model observed human behavior. This paper introduces the concept of a preference function, which is a generalization of the utility function. A preference function may be used to descriptively model actual human behavior. The paper also lists some of the key considerations in developing a preference function, including the combining of preferences of two or more parties. It provides some illustrative examples of how preferences functions may be used within investments and actuarial science.

Section 1: Background and Scope

1.1 Background

The concept of a utility function, which is a model representing the preferences of a consumer, was rooted in the work of moral philosophers from the 18th and 19th centuries and was later integrated into the field of economics. It has experienced many revisions, but it was formalized based on economic concepts proposed by Gèrard Debreu in 1954.[1] His two requirements were that the preference ordering must be complete and transitive. In mathematical terms: Completeness means given choices A and B, and a utility function u,

$$\mathrm{u}(A) > \mathrm{u}(B), \mathrm{u}(A) < \mathrm{u}(B)\ or\ \mathrm{u}(A) = \mathrm{u}(B).$$

Transitivity means given choices A, B and C, and a utility function u,

$$If\ \mathrm{u}(A) > \mathrm{u}(B)\ and\ \mathrm{u}(B) > \mathrm{u}(C), then\ \mathrm{u}(A) > \mathrm{u}(C).$$

Although Modern Portfolio Theory had been introduced by Harry Markowitz in 1952,[2] the utility function became a foundational concept used in the development of portfolio selection from choices on the efficient frontier and the mean-variance analysis[3] capital asset pricing. Although Debreu’s assumptions were logical, Amos Tversky and Daniel Kahneman, two behavioral psychologists, demonstrated that the classical utility model and underlying assumptions did not reflect actual human behavior.[4] For their work, three of these men received Nobel Prizes in Economics: Debreu (1983), Markowitz (1990) and Kahneman (2002). Had Tversky been alive, he may have been awarded the Nobel Prize with Kahneman, since they had jointly produced the recognized work.

1.2 Scope

The classical utility function is usually based on a normative approach and simplifying assumptions. A normative approach involves using underlying assumptions, which if correct can predict average or most common human behavior. The alternative is a descriptive approach, which attempts to model actual human behavior without necessarily understanding or modeling the underlying reasoning. Although a normative approach may be useful in many circumstances, it may not produce the best model for use in some areas of actuarial science. This paper explores some of the shortcomings of the classical, normative utility function and offers a more descriptive approach to modeling a measure of the desirability of outcomes. To distinguish the alternative measure of preferences from the classical utility function, the term actuarial preference function, or simply preference function, is used. The applications in the fields of investments, life insurance, and pensions are straightforward, and with some modifications the preference function may also be used in the accident and health or the property and casualty fields.

This paper is intended to address the concepts of a preference function within the context of actuarial practice and modeling. Some examples are provided as illustrations, and issues related to combining the preferences of different stakeholders and calibration of a preference function are considered. This is intended to provide anecdotal and conceptual foundations of an actuarial preference function together with some simplified examples for illustrative purposes; however, the actual development and implementation of actuarial preference functions will vary with the key parameters chosen and are beyond the scope of this paper.

Section 2: The Classical Normative Utility Function

Freidman and Savage[5] explained utility as the assignment of numbers to every level of wealth so that an individual (or consumer unit) will act to maximize expected utility in the face of known odds. Risk-seeking behavior occurs when the individual would accept a fair bet and risk-avoiding behavior occurs when the individual refuses to accept a fair bet. Markowitz expanded the utility function[6] and used it to determine how to optimize a portfolio[7].

2.1 Assumptions and Examples

The classical economic utility function maps a domain of wealth to a level of utility or use. The standard assumptions are:

- Utility is a function of or related to wealth;
- Utility is a monotonically increasing function, i.e., more is better; and
- Utility is continuous.

Additionally, the assumption of risk aversion or the law of diminishing returns is often added as a property of the utility function. It could be stated as:

- Utility has a negative second derivative, that is, each additional unit of wealth increases utility less than the prior unit of wealth.

The classical normative utility theory is derived from economic utility theory, which is based on several underlying assumptions about the rational behavior of consumers:

- People behave rationally,
- People have consistent preferences,
- Utility is a complete function, and
- Utility is a transitive function.

Some utility functions based on wealth include the exponential utility function

$$u(x) = \frac{1}{a}(1 - e^{-ax}), \qquad with\ a > 0;$$

the power utility function

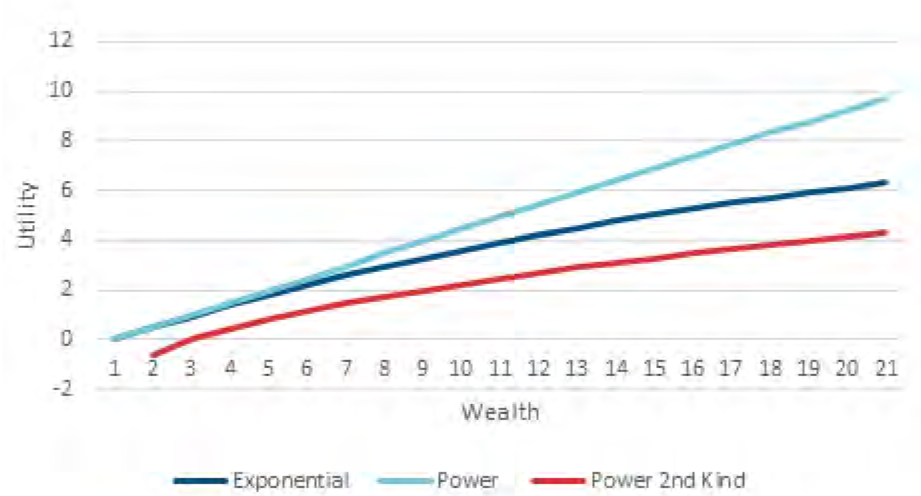
$$u(x) = \frac{s^{c+1} - (s - x)^{c+1}}{(c + 1)s^c}, with\ s > 0\ and\ c > 0;$$

and the power utility function of the second kind

$$u(x) = \frac{x^{1-c} - 1}{1 - c}.$$

These utility functions were surveyed in an actuarial context by Gerber and Pafumi.[8] A graphical depiction of them using a=0.1, s=100 and c= 0.5 is shown in Figure 1.

Figure 1
SOME UTILITY FUNCTIONS



For the functions described in the following paragraphs, r_p denotes the return of a portfolio, r_f denotes the risk-free rate, and β denotes the correlation coefficient between the portfolio and the market. Instead of wealth, the associated functions measure the desirability of a combination of expected returns and the variability of the returns.

Several functions, which incorporate the measures of risk and return to provide sets of comparative and equivalent values based on the combinations of the two metrics, have been used in the field of investments. Since they are not a function of wealth and do not have diminishing returns, they are not utility functions in the strictest sense of the definition. However, they offer a means of determining the preferences of sets of possible returns. They could be considered a set of investment preference functions. They are normative functions and share several characteristics with utility functions. They are based on some reasonable assumptions:

- All other things being equal, people will prefer a higher expected return to a lower expected return.
- All other things being equal, people will prefer the option with the least amount of risk, which is defined in terms of the uncertainty of some or all of the returns within the distribution of possible returns.
- The function measuring desirability of a return and risk is complete.
- The function is transitive.
- The function is continuous with respect to return and with respect to the risk measure.

Probably, the most familiar and commonly used function is the Sharpe ratio, which is the expected return of an investment less the risk-free rate of return divided by the standard deviation of return on the investment. It was used by Markowitz in the development of modern portfolio theory using mean-variance optimization.

$$Sharpe\ Ratio = \frac{Exp(r_p - r_f)}{StdDev(r_p)}$$

One of the disadvantages of the Sharpe Ratio is that excess returns that alternate between 10% and 12% (i.e., Sharpe Ratio = 0.11/.01 = 11) would have a lower ratio than an excess return that alternated between 8.01% and 7.99% (i.e., Sharpe Ratio = 0.08/0.0001 = 800). Another disadvantage is that noninvestment professionals often consider risk as the possibility and magnitude of a loss or less than anticipated return instead of the standard deviation of return. The Sharpe Ratio can be generalized to include a factor denoting the the level the standard deviation of returns impacts the investor's choices. Using *x* as the factor for the level of weight given to the standard deviation of returns, this generalization is

$$Generalized\ Sharpe\ Ratio = \frac{Exp(r_p - r_f)}{StdDev(r_p)^x}.$$

The same risk-weighting approach could be applied to many other standard investment preference functions.

The Sortino Ratio is an alternative measure of investment preferences, which only considers the volatility of losses as the risk. The return target, denoted r_t, can be set at 0 or any chosen level.

$$Sortino\ Ratio = \frac{Exp(r_p - r_f)}{StdDev(r_p | r_p < r_t)}$$

Return over Maximum Drawdown (RoMaD) is measure of investment preferences often used in evaluating hedge funds, since it compares the expected return to the greatest expected loss. It uses the generally understood idea of risk being a measure of the worst-case scenario or the greatest decline of all high values to a subsequent low value.

$$RoMaD = \frac{Exp(r_p - r_f)}{Greatest\ Percent\ Decline\ in\ Value}$$

Alpha is a measure of investment performance. It is the excess return of a portfolio above the expected return based on the systemic risk assumed. The total return of a portfolio r_P is rated to the return of the benchmark market index r_M based on the formula:

$$r_P = \alpha + \beta r_M + \varepsilon, \qquad \text{where } \varepsilon \text{ denotes the error or "noise" in the results.}$$

Since this is a measure of the portion of return attributed to a portfolio manager's skill, it is often used to evaluate portfolio managers and mutual funds. Alpha is usually measured retrospectively and assumed to be a proxy for the immediate future alpha of a fund manager.

$$\alpha = Exp\left(r_p - (1-\beta)r_f - \beta(r_m)\right)$$

The Treynor Ratio is similar to the Sharpe Ratio, but uses the systemic risk, or the market-based portion of the portfolio's risk, to measure volatility instead of using the total risk.

$$Treynor\ Ratio = \frac{Exp(r_p - r_f)}{\beta} = Exp(r_m) + \frac{\alpha}{\beta}$$

These functions are easily manipulated and modeled and can be used to measure or improve the expected outcomes for many situations. They are based on a normative approach, which is based on some assumed axioms and prediction of how an ideal agent would behave if he or she were to follow the axioms. An alternative is the descriptive approach, which is intended to describe how things actually happen. Often a normative approach is used to derive functions applied to models that are by their nature intended to be descriptive. Actuaries frequently combine the two approaches, using normative approaches to approximate interpolations based on descriptive information, with periodic refinements based on improvements in the normative approaches and the descriptive data. Jordan[9] showed this in his description of the historical development from DeMoivre's approximation of the number of lives being linear from about 12 to 86, through Gompertz's assumption of "the average exhaustion of a man's power to avoid death"[10], through the application of Makeham's law[11] used in the construction of statutory mortality tables since the early 20th century, to the select and ultimate mortality tables used today with various assumptions for interim mortality (e.g., uniform distribution, constant force of mortality, Balducci distribution).

2.2 Advantages and Uses of the Classical Utility Function

The classical, normative utility function approach offers many advantages. It provides a simplified model of human behavior and can be used to reasonably reflect the aggregate behavior of consumers or other economic units. It is often represented by a compact mathematical formula, providing a simple, easily manipulated model. Despite the simplicity of the models, they offer understanding and insights relative to the concepts of utility and human behavior, and they frequently improve the predictive results for many problems. Some clinical experimentation has shown that the use of a simple, calibrated actuarial model generally produces results at least as good as those produced by trained clinicians. Lewis Goldberg, who has written on clinical and actuarial judgment, is reported to have commented, "[O]ver a rather large array of clinical judgment tasks (including by now some which were specifically selected to show the clinician at his best and the actuary at his worst), rather simple actuarial formulae typically can be constructed to perform at a level of validity no lower than that of the clinical expert."

Another advantage to using the classical, normative utility function is that it is generally accepted and thus more conservative than using an alternative. They are usually based on assumptions which are logical and can be justified using reason. Unexpected results may be attributable to errors in the function rather than requiring justification for its application. Employing a generally accepted concept is easily defended. Undertaking the application of an unproven approach may require much more justification and increases the risk of failure; however, risk of failure is the cost of innovation.

The popularity of classical utility functions has resulted in familiarity with their advantages and uses by those who employ them. More attention has been given to their limitations. The degree to which this paper explores their restrictions and the limited acknowledgment of their advantages should not be considered a dismissal of their value and practicality. Despite potential shortfalls, a classical utility function may often provide a suitable model for the particular circumstances considered.

2.3 Restrictions of the Classical, Normative Utility Function

The normative approach only reflects an idealized model of reality. Under some circumstances, varying from the traditional assumptions may be warranted to produce an improved model. We will consider some of the assumptions of the classical, normative utility function and note observations of circumstances when these assumptions may not be appropriate in modeling preferences. We will also explore how these might be actuarial considerations. Each of the assumptions discussed below may or may not be appropriate to apply to a particular preference function.

2.3.1 People Behave Rationally

This comes into the normative portion of the classical, normative utility function. Normal human behavior is assumed to be rational, and any individual who does not behave rationally is considered an outlier. However, in some ways people tend to consistently behave irrationally.

If people truly behaved rationally, why would the same person buy insurance, a risk-avoiding behavior, and participate in a lottery, a risk-seeking behavior? This question was addressed by Kahneman and Tversky. They found that people tend to overweight small probabilities. If a person can perceive something as possible, it becomes an emotional or psychological probability.

People have cognitive biases based on presentations. Depending on how a choice is offered can impact the preference. Kahneman and Tversky used the following experiment:

> A person was given \$1,000 and given the option of an additional \$500 or, based on the flip of a fair coin, either \$1,000 or \$0 additional. Most people chose the additional \$500. An alternative was a person was given \$2,000 and given the option of paying \$500 or, based on the flip of a fair coin, losing \$1,000 or \$0. In the second case, most people chose to take the risk of the outcome from the coin toss.
>
> First Scenario: $u(\$1,000 + \$500) = u(\$1,500) > 0.5 \times u(\$1,000+\$1,000) + 0.5 \times u(\$1,000+0)$ $= .5 \times u(\$2,000) + 0.5 \times u(\$1,000)$.
> Second Scenario: $u(\$2,000 – \$500) = u(\$1,500) < 0.5 \times u(\$2,000 – 0) + 0.5 \times u(\$2,000 – \$1,000)$ $= 0.5 \times u(\$2,000) + 0.5 \times u(\$1,000)$ for the same person, which is a contradiction to the utility of the first scenario, since the two combine for $u(\$1,500) > 0.5 \times u(\$2,000) + 0.5 \times u(\$1,000) > u(\$1,500)$.

The expected outcome and thus the expected utility of the two experiments was identical if based on wealth. However, the difference in preferences was based on the perception of the change in condition. Similarly, when patients considering a surgical procedure were told they had a 90% chance of survival, they were much more likely to opt for the procedure than if they were told they had a 10% chance of death.

People often confuse correlation and causation. This is part of our instinctive survival mechanism. We seek to discern patterns and have difficulty accepting randomness where we discern a pattern or a pattern where we discern randomness. This can lead to making large assumptions based on unjustifiably small samples or unreasonable expectations.

People place too much reliance on associations or similarities. When told, “Fido barks and has a natural tendency to chase cars,” and then asked, “Is it more likely Fido is a dog or an entity in the universe?” most people indicated it was more likely that Fido was a dog. The association with the name Fido, barking and chasing cars led them to mentally imagine a dog and not think of every dog being an entity in the universe. Similarly, if asked which outcome is more likely if 20 marbles are randomly distributed to Abe, Ben, Carl, Dan and Evan, people tended to choose 4, 4, 5, 4, 3 over 4, 4, 4, 4, 4.[12] Although the uniform distribution is the more likely, the expectation of some randomness led them to choose the non-uniform distribution.

People overrate pain and underrate gain. A negative experience has a much more profound impact on a person’s memory than a positive experience. We tend to remember the one time we touched a hot stove more than any of the hundreds of times we touched a cool stove. This is part of our instinctive survival mechanism. The more severe the negative experience, the more memorable it becomes. Avoiding damage is key to survival and goes beyond the physical to the emotional and financial aspects of life as well. Often the attempt to avoid a small financially adverse outcome can lead to a greater adversity later. Delays in modest rate increases can lead to larger rate increases later and provoke reactions from policyholders, which could have been avoided through smaller incremental rate increases. In the previously described experiment where people were given the option of a certain amount or a 50-50 chance of $1,000, Kahneman and Tversky found that if someone was neutral between the 50-50 chance of $1,000 or $0 or the known gain of $370, they would be willing to pay even less than $370 to avoid a 50-50 chance of a loss of $1,000 or no change. They found that in general people tend to be risktakers when facing a loss and risk averse when facing a gain.[4] Insurance is feasible because it provides a preferred combination of economic utility and psychological benefit to the consumer over the economic and psychological present value of the premium payments, and because it also provides an expected economic gain to the insurer. The adage, “Insurance is sold, not bought,” is a reflection of the importance of the psychological component of the decision to buy insurance.

People tend to be defensive or cognitively conservative, meaning they will adhere to a decision or idea they have accepted even when they have been presented evidence to the contrary. In fact, the stronger the initial acceptance the more they tend to hold to their prior conviction.[13,14] This is called the backfire effect. Similarly, people tend to interpret, believe and remember information that supports their preexisting beliefs.[15] This is called confirmation bias.

The recognition of these human cognitive biases can be instrumental in product design, marketing, pricing and measuring the satisfaction of policyholders, stockholders and other stakeholders in developing a model of their preferences.

2.3.2 Utility Is Continuous

Money is in discrete units, so continuity of utility involving money is a simplification, albeit, a generally reasonable simplification. However, there are circumstances that provide quantum states of utility or preferences. These include solvency vs. bankruptcy, action levels based on risk-based capital levels, rating levels issued by rating agencies, contribution rates for pensions (when rounded to 0.1% or 0.25% of salary intervals), the addition of a new employee and the addition of a new product line. In the case of jumps in contribution rates, the preference levels may rise with increases in the funding levels, but have discontinuous leaps in preference as the funding level crosses a contribution rate threshold. When constructing a preference function, these could best be modeled using a discrete function or one that is discontinuous at the quantum changes. The binary preference function developed in section 4.1 provides a simple example of such a discrete function.

2.3.3 Utility Is Monotonically Increasing

Although the economic idea that more is better is generally true, there are limits, beyond which utility or preference begins to decrease. One way to envision this is to consider an extreme: few people would actually want to own the entire world. At some level the responsibilities from the gains can be less desirable than the benefits of the gains. An extremely overfunded pension plan can result in greater benefit demands from labor, increased benefits and future obligations, decreased retention of good employees due to earlier retirements, and added governmental scrutiny and potential excise taxes. Too much profit encourages excessive competition, potential demands for wealth redistribution accompanied by control being wrested away and increased scrutiny and restrictive laws to the detriment of the profitable company.

2.3.4 Utility Is Translational and Tends to Be Isolated in Time

Most utility functions assign a utility based on a current state of wealth or return. The concept of measuring utility based on return does include a comparison of the beginning and ending states during a period; however, it does not include the total path, or sequence of states. The desirability of a state depends on the sequence of previous states and the relation between the current state and the expectation for the current state. The satisfaction an individual will derive from $1 million in their bank account depends on how much they previously had in their bank account and how much they expected to have in their bank account. If yesterday they had $1,000 and were expecting $1,000, they would be more satisfied than had they had $5 million in the account yesterday and were expecting $5 million today. Similarly, the desirability of a 4% return will depend on recent historical returns of similar investments, the current returns of similar investments and the expectations of the investor. Since people and the markets tend to overweight losses and underweight gains, an insurance company that has its rating changed from B to A and then back to B over a short period would generally be less favorably impacted than had it maintained a B rating. This is often reflected in the generally modest way the stock market reacts when companies exceed earnings expectations as compared to the generally more severe reaction of the market when companies fail to meet earnings expectations.

One of the traits of evaluating a path in terms of preference is that the further a state is from the moment being evaluated, which is typically the present, the less impact it has on the expectation and therefore the preference. With time, often bad gets better and good gets worse. Bankruptcy within the year is a greater concern than within 10 years and bankruptcy within 10 years is a greater concern than within 100 years. Likewise, a bankruptcy 100 years in the past is expected to have less impact than a bankruptcy 10 years in the past; and both of these can be expected to have less impact than a bankruptcy in the past year. A significant excess return in the past quarter garners a greater positive response from investors than the same excess return in the prior business cycle; and a significant excess return in the prior business cycle garners a greater positive response than the same significant return having occurred several decades earlier.

Expectations are also driven by the assumption of fairness. Individuals expect the same results they observe to be experienced by others. The concept of fairness is a part of our primitive instincts and has been observed in other mammals and in birds. Two capuchin monkeys were given a task to perform and were rewarded with cucumber slices. They were content; however, when one of the monkeys was rewarded with grapes instead in the presence of the other monkey, the monkey who was rewarded with cucumber slices began to shake the cage, slap the floor and would throw the cucumber slices at the person performing the experiment.[16] We have a primal drive for fairness and to benchmark ourselves based on our perception of others we consider similarly situated. One example where this fairness principle is sometimes used in actuarial science is in the development of dynamic lapse rates. If a company's competitors are offering a higher crediting rate than the company, the policyholders become more likely to exchange their policies for policies with the higher crediting rate. This dynamic lapse rate being dependent on what the policyholders can get elsewhere is a reflection of the preference based on comparison with others similarly situated.

The preference of a state is based on prior states, stability of the states, expectations of the current state and comparisons with others perceived as similarly situated. The classical, normative utility function frequently fails to capture these aspects of a consumer's preferences.

2.3.5 Utility Is Transitive

The intransitivity of preferences has been noted in the selection of a potential spouse when various combinations of attractiveness, wealth and intelligence were presented.[17] I am unaware of any insurance-related studies showing intransitivity in product selection preferences. A preference function that maps the preferences of a state or path to a real number will be transitive. Although a preference function may be intransitive and could be of theoretical interest, we will only work with examples which are mapped to real numbers and are therefore transitive.

Section 3: Actuarial Considerations in Developing Preference Functions

3.1 Definition of a Preference Function

The concept of a preference function is a generalization of the utility function. One may choose to use a preference function when the restrictions of utility functions are an obstacle to providing a model of the actual preferences or observed behavior. The choice of which to use is based on the trade-off between ease and accuracy. Often a utility function can reflect reality accurately enough to employ in a model.

A preference function is a function that provides an ordering of the comparision of choices. A preference function on a finite set of choices can be depicted by using a complete directed graph. The complete directed graph of a preference function could be defined for a set of finite choices C by the mapping P(C) = {(x,y) | x,y ∈ C and y is not more preferable than x}. Since the graph is complete for every pair of elements {x,y} in C, at least one of the ordered pairs (x,y) and (y,x) are elements of P(C). The complete directed graph as a preference function is an application within graph theory, a branch of discrete mathematics, and will not be developed in this paper. The preference function could be expanded to infinite sets using the same definition.

Preference functions can be divided into two categories, preference functions that are utility functions and those that are not utility functions. A preference function that is not a utility function will be referred to as a generic preference function.

We will be considering a subset of preference functions that assign a real number to the choices. If p is such a preference function, then $p:C\rightarrow\mathbb{R}$, with x preferred to y if and only if $p(x)>p(y)$. This collection of preference functions that assign real numerical values to the preferences of a choice will be transitive.

3.2 Advantages and Limitations of Preference Functions

3.2.1 Advantages

Preference functions come from Prospect Theory, which has surpassed Utility Theory in many fields of practice, including medicine, psychology, politics and economics. Its application appears to be the next milestone in modeling human behavior and it provides a better reflection of reality than a utility function derived solely from logical assumptions. It is based on descriptive preferences instead of assuming people behave in a normative, rational manner. Opportunities to insure risks exist within the gaps between purely logical economic decisions and behaviors based on preferences.

Preference functions are more diverse and less restrictive than utility functions. Utility functions are a subset of preference functions. Preference functions can be designed to reflect the preferences of various paths of states and could be applied to futurism in ways that better reflect reality than the classical utility functions.

Preference functions can combine psychological and economic components to reflect likely behavior of consumers in areas where insurers and businesses can benefit economically by providing psychological and economic benefits to individuals or small groups.

3.2.2 Limitations

Since preference functions are expected to describe behaviors, their derivation may require extensive studies to properly model human preferences. Preferences may be difficult to model, justify and explain, because they are sometimes counterintuitive. Instead of valuing a state, preferences may assign values to changes in states, which is the basis of decisions; however, this adds a level of complexity to the concepts and models used.

For the sacrifice of each assumption used in the development of a utility function, we lose valuable mathematical tools available for analysis of the results. The incorporation of a generic preference function into a model may adversely impact the ease with which the output can be studied through arithmetic manipulation and may require something as computationally intense as Monte Carlo testing to study the results.

3.3 Identifying the Stakeholders

One of the key questions in developing a preference function is to determine whose preferences are being considered. In economics the concept of demand is related to the price an economic unit is willing to pay for a good or service. If the economic unit has no money, it has no demand. Often stakeholders are considered those who are impacted by a decision; however, in considering stakeholders for a preference function, the key identifying characteristic of a stakeholder is the ability of the stakeholder to accept or reject what is preferred. Although insured lives and beneficiaries of insurance policies may be impacted by the policy provisions and the company's execution of the provisions, it is the policyholders who have the power to accept or reject the policy offering. Company owners can decide whether to invest in the products offered by the company and therefore are stakeholders. The Board of Directors and management can decide whether to offer a product. Agents can decide whether to sell a product. Regulators can decide whether to permit a product to be sold. Each of these could be stakeholders for consideration. They may be considered individually or in aggregate. Frequently the different stakeholders are considered separately and a feasible solution is designed as a compromise.

3.4 Identifying the Key Preference Drivers

To identify the key preference drivers of a stakeholder the actuary may use his or her professional judgment and historical experience, sample representatives from the various stakeholders or perform a Delphi study. We will consider some of the potential key drivers for a stock-owned life insurance company by stakeholder. The lists are not intended to be exhaustive. Utility functions are subsets of preference functions and reflect the economic condition of an economic unit. A preference function can be based on the preferences of one or more stakeholders.

3.4.1 Company Owners—Stockholders

Company owners provide capital for the operation of the business and expect reasonable profits in return. The expected return on capital is often called the hurdle rate. Just as spot interest rates tend to increase with time, if the realization of profits is deferred, the return is generally expected to be greater. Owners expect managers to efficiently use the capital entrusted to them. The internal rate of return is the optimal rate for a business to grow. If it grows faster, additional funds may be required through loans or capital. If the company grows its business slower than the internal rate of return, then the company's surplus increases faster than its assets and liabilities. Excess surplus is expected to be distributed through dividends or stock repurchases unless the company can efficiently use the funds to generate additional profit. Owners preference functions could include considerations of profits, stability, dividends and/or growth.

3.4.2 Company Management

Company management is concerned with the detailed operations of the business. A preference function for management may include risks and how they are mitigated. Areas of concern include profits, financing, reinsurance operations, expenses, underwriting, competition, pricing, crediting strategies, investments, the impact of capital requirements, solvency, operational control, reputation, and market share.

3.4.3 Regulators

Regulators have the function of protecting the public. This is done through taxes intended for the public benefit, ensuring insurers treat policyholders fairly, ensuring there is competition within the insurance markets and protecting the solvency of insurance companies. The preferences of regulators or government in general would include taxes and revenue, market conduct, competition, solvency and compliance.

3.4.4 Agents

Insurance agents are compensated from the sales of insurance products. Their preferences would include greater commissions on policies, commissions paid over a longer period of time, that policies be serviced, that sales have higher volume and greater ease, that products are competitive in the market, and that they are provided sales support.

3.4.5 Policyholders

Policyholders pay premiums in exchange for promised benefits. They prefer lower prices, richer benefits, clear understanding of the policies, policy service and security that the promised benefits will be paid as understood. As consumers they frequently compare policies with the competition and wish to minimize the cost of exiting a policy.

3.5 Including the Factor for Time

Time impacts preference in two ways; one is that the amount of time from the present or moment being evaluated generally reduces the impact of the preference. The other is that time is viewed as the sequencing of events, and since the order of changes impacts the desirability of a state, the timing of the states impacts the preference of a given state.

Several options are possible for the discounting of the impact of time. One approach is to assume a future event is discounted at the risk-free rate and past events are discounted at the mirrored current risk-free rate or the historical risk-free rate. Since risk-free rates change with the period being considered, there is typically an acceleration of the discount. The use of risk-free rates may be an appropriate representation of the economic value, but it often does not reflect the preferences of individuals or economic units. The fact that people will often accept settlements for significantly less than the economic value is an indication of this divergence in preference from economic value. Some specific examples include workers' compensation settlements, cash payments for annuities or structured settlements, and viatical settlements. The economic discount of value would be defined by the formula

$$Discounted\ economic\ value\ of\ P\ at\ time\ t = P\ e^{-r_f(t)\times t}.$$

This formula implies past value (i.e., $t<0$) is greater than current value. This is because a level of wealth in the past could have been invested to produce a greater level of wealth in the present, so the economic value increases with time. However, when considering preferences the present state is the most important. The importance and impact of the past and concern about the future both diminish as they are more distant from the present. To discount past value for preference functions, we can take the absolute value of the time. If we consider a normal spot rate yield curve, a reasonable approximation of it is

$$r_f(t) = k \times \ln(1+t)\,, for\ t \geq 0.$$

If an economic unit x uses a similar discount curve, the discount for preference can be estimated using

$$r_x(t) = k_x \times \ln(1+t)\ \ for\ the\ discount\ rate$$

and

$$Discounted\ preference\ of\ P\ at\ time\ t = P\ e^{-r_x(t)\times|t|} = P\ \times \left(\frac{1}{1+|t|}\right)^{k_x\times|t|}.$$

The factor k_x would be determined by estimation or calibration and may be different for the t<0 and t>0, depending on how the individual weights the past and the future. It is not unreasonable to expect that a person would use a smaller discount factor for the past than the future, since the past is known and therefore has no uncertainty related to it. If k=0 for a period of time (e.g,. for the past or t<0), then the period of time would have no impact on the current preferences. The preferences of the period would be just as important as the current preference. This would be the preference equivalent of the financial condition of a zero interest rate environment. If k<0 for a period of time, then that period of time would have greater impact than the present and would be the preference equivalent of the financial condition of disinflation or a negative interest rate. If a period of time has no bearing on the current preferences, then that period could be ignored or for that period k could be considered infinite.

The other consideration of time is the change that occurs in it. In economics the standard return is the risk-free rate, which is a change in wealth. When considering preferences, the standard for comparison is expectations. Expectations can be impacted by past experiences, comparisons to others similarly situated, the information available and the interpretation of the available information. As expectations change the preferences will change. As experience unfolds, expectations tend to revert to recent past experience and the experience of others similarly situated. Preference is, therefore, also a function of time and the path of states.

Since losses are punished more than gains are rewarded, stability, or consistently achieving expectations, through time tends to increase preferences.

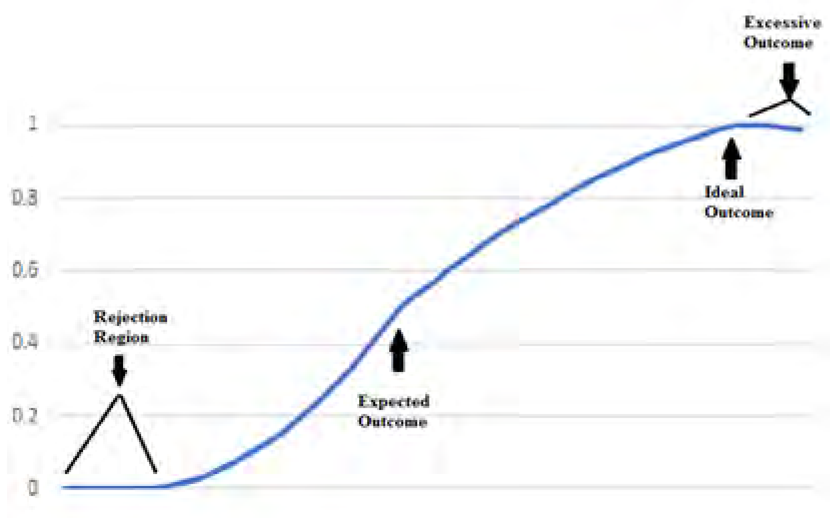
3.6 The Range of a Preference Function

Like a utility function, which is a subset of preference functions, the preference function has a domain of the set of choices or states. The function maps each element of the domain to a real number. By selecting certain benchmarks, a preference function can be calibrated to provide some information about the desirability of outcomes. Although the range and calibration may be assigned by the modeler, the following range and calibration points provide an example of how the results can be interpreted.

$$P(x) = \begin{cases} 1 \; if \; x \; is \; ideal \\ 0.5 \; if \; x \; is \; what \; is \; expected \\ 0 \; if \; x \; results \; in \; rejection. \end{cases}$$

This formula indicates preferences above 0.5 as favorable and below 0.5 as unfavorable relative to expectations. To include the greater impact of adverse experience, one may modify the results between 0 and 0.5 to be concave and 0.5 and 1 to be convex. Figure 2 indicates some level the results are rejected, then there is a range of outcomes where risk is preferred over the expected conditional adverse outcomes. Once the expected outcome is achieved, risk is avoided when given conditional favorable outcomes. Finally once the ideal outcome is achieved, more is considered less preferable. In actual practice the excessive outcomes may be considered as ideal.

Figure 2
A PREFERENCE FUNTION



Using the formula presented above, let us consider RBC ratios as they may be viewed by a Board of Directors and as they may be viewed by regulators. The Board of Directors has set a target RBC ratio of 10. They believe a higher ratio indicates the capital could be put to better use and a lower ratio could adversely impact the company's ratings. The recent annual RBC ratios have been 7.2, 7.4 and 7.5. Given the current trend, the Board expects the next RBC ratio to be 7.65. Since the regulators can take action if the RBC ratio is 1.0 or lower, the Board would reject any results that produce an RBC ratio of 1.0 or lower. The preference function for RBC levels by the Board of Directors for the next year may be represented by

$$P_{Board}(RBC) = \begin{cases} 0.25 + 0.75 \times \left(\frac{10}{RBC}\right), for\ RBC \geq 10 \\ 0.5 + 0.5 \times \frac{(RBC - 7.65)}{(10 - 7.65)}, for\ 7.65 \leq RBC < 10\ and \\ 0.5 \times \left(\frac{Max\{RBC, 1\} - 1}{6.65}\right)^2, for\ RBC < 7.65 \end{cases}$$

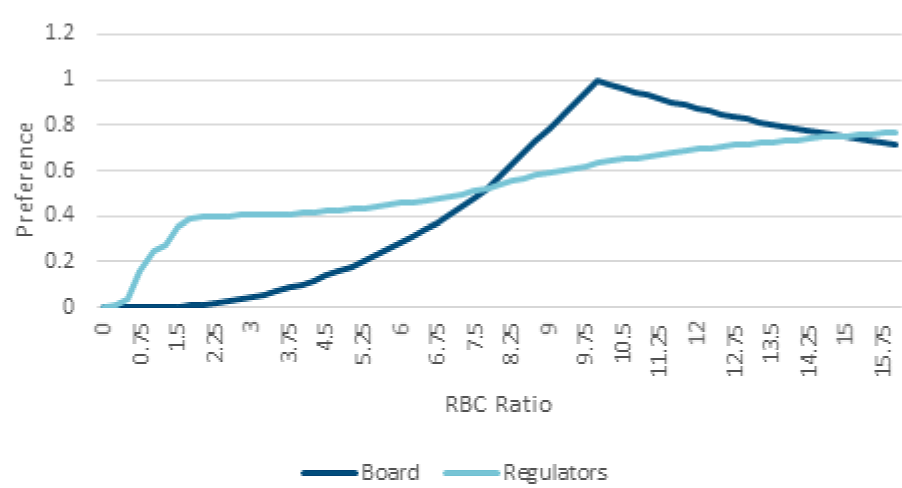
The regulators tend to view more capital as always being more favorable, so the ideal would be unlimited capital. The unacceptable level for the regulators is insolvency or an RBC ratio 0. They will be required to take control at an RBC ratio of 0.7 or lower, may take control at an RBC ratio of 1.0 or lower, must take action at an RBC ratio of 1.5 or lower and require company action at the RBC ratio of 2.0 or lower. And the regulators may expect the RBC ratio to be the average of recent RBC ratios, or 7.37. The preference function for the regulators may be represented by

$$P_{Regulators}(RBC) = \begin{cases} 1.0 - 0.5 \times \left(\frac{7.37}{RBC}\right), for\ RBC \geq 7.37 \\ 0.4 + 0.1 \times \left(\frac{RBC - 2}{5.37}\right)^2, for\ 2.0 \leq RBC < 7.37\ and \\ 0.35 + 0.05 \times \frac{\sqrt{RBC - 1.5}}{0.707}, for\ 1.5 \leq RBC < 2.0\ and \\ 0.25 + 0.05 \times \frac{(RBC - 1)}{0.5}, for\ 1.0 \leq RBC < 1.5 \\ 0.15 + 0.05 \times \frac{(RBC - 0.7)}{0.3}, for\ 0.7 \leq RBC < 1.0\ and \\ 0.1 \times \left(\frac{Max\{RBC, 0\}}{0.7}\right)^2, for\ RBC < 0.7 \end{cases}$$

The regulators preference function has discontinuities at various levels of required action. Figure 3 compares these two preference functions. The discontinuities are depicted in the graph as steep jumps rather than gaps.

Figure 3

BOARD AND REGULATOR RBC PREFERENCES



Each decision maker has their own preference function. Unless the preference functions of two different decision makers are calibrated to each other, their preferences cannot be compared. So at an RBC of 9, the regulators may actually have a higher satisfaction level than the board, although the graph indicates a higher number for the board's preference. If the two are calibrated they can be combined to determine an overall preference. This may be done several ways, including a weighted linear combination and a weighted multiplicative combination. When combining preference functions in the following examples, we will assume the preferences of various stakeholders have been calibrated.

Section 4: Illustrative Examples of Actuarial Preference Functions

Although preference functions are used in the following examples, it may be possible to reasonably approximate a preference function using a utility function over the range of reasonable outcomes. The mathematical tractability of utility functions could make the utility function approximation preferable. To the degree possible, one may wish to ascribe one or more of the properties of a utility function to the preference function used to aid in the future analysis.

4.1 Binary Preference Function

One of the simplest preference functions is the binary preference function. For a given state or path, the preference is either acceptable or not acceptable. It can be defined as

$$P_{binary}(X) = \begin{cases} 1\ if\ X\ is\ acceptable \\ 0\ if\ X\ is\ unacceptable. \end{cases}$$

For an illustrative example, suppose the requirement for an acceptable outcome is that an investment be doubled in 10 years and anything less is unacceptable. The investment options are equities, long-term bonds, and US Treasuries. How would one optimize the expected preference of the outcome?

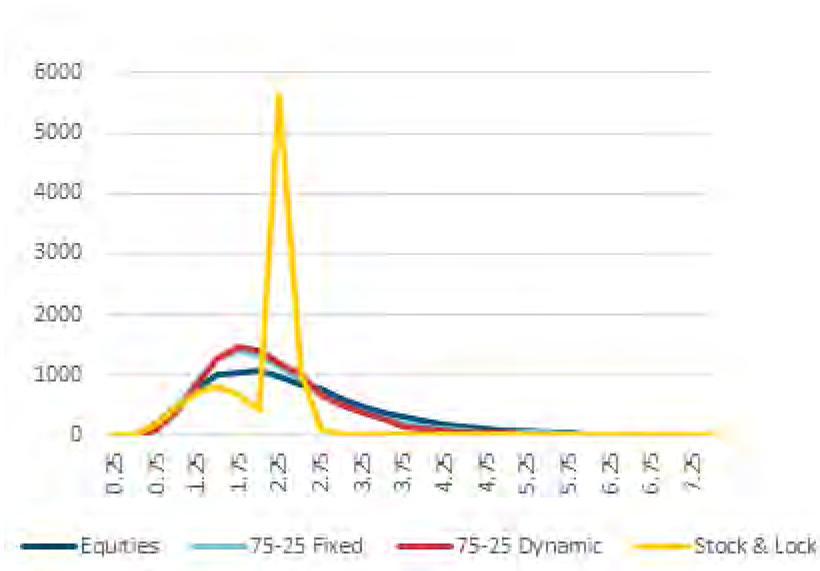
Since equities have the highest expected return, investor A chose to invest 100% in equities. Investor B chose to invest 75% in equities and 25% in bonds without altering the allocation. Investor C chose to use the same allocation but reallocated the assets each year. Investor D chose a "stock and lock" approach, in which he invested 100% in stock and then switched to Treasuries at the end of any year in which the Treasuries would ensure the final value was twice the starting value.The outcomes of 10,000 randomly generated scenarios, produced by the SOA's Actuarial Interest Rate Generator (AIRG 7.1) are provided in Table 1.

Table 1
OUTCOMES OF VARIOUS INVESTMENT STRATEGIES

Investor/Method	Goal Achieved	Goal Achieved then Lost	Average Wealth	Standard Deviation
A—100% Stock	5,429	573	2.330	1.152
B—75/25 Fixed	4,591	442	2.087	0.883
C—75/25 Dynamic	4,547	387	2.042	0.784
D—Stock & Lock	6,788	0	1.879	0.445

The distributions of the returns are depicted in Figure 4.

Figure 4
DISTRIBUTIONS OF RETURNS



Of the four strategies, the strategy selected is based on the preference function used. The 100% stock portfolio had the highest expected return, so it would be the preferred strategy if highest return determined the preference of the investor. The stock and lock offered the greatest expected return for the risk assumed; however, it provided the lowest overall expected return and the standard deviation of the returns does not reflect the skewness of the return being concentrated around wealth factor of 2. The 75/25 dynamic allocation produced the expected wealth factor of at least 2 with the minimum amount of standard deviation of returns, so it would be the preferred strategy if the preference was based on minimizing the standard deviation of returns given an expected wealth factor of 2. Using the binary preference function described initially, the greatest expected preference is from the stock and lock, which is 0.6788. This illustrates both the binary preference function and how different preference functions can produce different optimal investment strategies.

For the interested reader, Chen and Hieber[18] studied normative optimal asset allocation in life insurance considering the impact of regulation.

4.2 Employing a Preference Function for a Defined Benefit Pension Plan Investment Decision

R. J. Thomson wrote extensively on the use of utility functions for investments in defined contribution retirement plans to address the concerns of the plan sponsors and participants.[19] For an illustrative example, we will consider the preferences for a defined benefit plan.

Suppose the Board of Trustees of a Public Defined Benefit Plan are considering changing their investment strategy from a 25% equity and 75% bond portfolio to 50% equity and 50% bonds. The investment adviser has explained that the expected return on bonds is 5% with a quarterly standard deviation of 2% and equities have an expected return of 9% with a quarterly standard deviation of 6%. He explains how the quarterly risk-free rate is 1.0% and the Sharpe ratio of the proposed change is 1.34 compared to the 1.44 ratio for the current investment strategy. He advised the Board not to change its investment strategy since the current portfolio offered a higher return for the risk assumed. The Board of Trustees asked the pension fund's actuary to also advise them on whether to make the change.

The actuary asked the board members to use a scale of –10 to 10, with 0 being neutral and –10 being completely unacceptable and 10 being ideal, to rate various scenarios. The average results are provided in Table 2:

Table 2
SAMPLE RATINGS OF A PENSION FUND'S BOARD MEMBERS' PREFERENCES FOR CHANGES IN CONTRIBUTION RATES

Scenario	Average Rating
An annual increase in contributions of 0.75%	–8.0
An annual increase in contributions of 0.50%	–6.2
An annual increase in contributions of 0.25%	–4.3
No change in the annual contribution rate	0.5
An annual decrease in contributions of 0.25%	3.0
An annual decrease in contributions of 0.50%	5.1
An annual decrease in contributions of 0.75%	7.0

The current valuation rate and expected portfolio return are 6% and the new portfolio strategy would change the expected return to 7%. The actuarial value of assets is determined using the market value of equities and average of market and book values for bonds. Since the contribution rates are set annually, the standard deviation used for equities is 12% and for bonds 4%. The 4% for bonds was derived using the market portion of the bond variance for 4 quarters or

$$\begin{aligned} &Std.Dev.of\ Actuarial\ Bond\ Return \\ &= Portion\ of\ Return\ from\ Market \times \sqrt{No.of\ Quarters} \times Quarterly\ Std.Dev. \\ &= 0.50 \times \sqrt{4} \times 2\% = 2\%. \end{aligned}$$

The standard deviation of return based on the actuarial value of assets is 6.25% based on the current portfolio and 8.60% based on the proposed portfolio. The actuary then considers how the two portfolios will affect funding. The amounts in Table 3 are provided for illustrative purposes and are the derivations depending on the amount and timing of benefits, salaries and other factors not provided.

Table 3

DERIVATION OF IMPACTS OF ONE STANDARD DEVIATION IN RETURNS FOR TWO PORTFOLIO OPTIONS

	Current Portfolio 6% Discount	Proposed Portfolio 7% Discount
Present Value of Future Benefits	42.09 million (future benefits discounted at 6%)	38.26 million (future benefits discounted at 7%)
Actuarial Value of Assets	19.10 million	19.10 million
Present Value of Future Contributions	42.09 million – 19.10 million = 22.99 million	38.26 million – 19.10 million = 19.16 million
Present Value of Future Salaries	239.04 million (future salaries discounted at 6%)	222.17 million (future salaries discounted at 7%)
Expected Contribution Rate	22.99 ÷ 239.04 = 9.62% (rounded to 9.5%)	19.19 ÷ 222.17 = 8.62% (rounded to 8.5%)
One Standard Deviation of Return on Actuarial Value of Assets	6.25% of 19.10 million = 1.19 million	8.60% of 19.10 million = 1.64 million
Contribution Impact of One Standard Deviation in Returns	1.19 ÷ 239.04 = 0.50%	1.64 ÷ 239.04 = 0.69%

The actuary then projects the potential returns and contribution rates (including an amortization for the change in assumed discount rate) and assumes 1/3 each for the probability of the expected contribution rates and rates one standard deviation above and below the expected rates. The results are summarized in Table 4 and Table 5:

Table 4

CURRENT PORTFOLIO MIX OF 25% EQUITIES AND 75% FIXED INCOME

Year	Lower Range	Preference	Mid-Range	Preference	Higher Range	Preference	Expected Preference
1	9.5%	0.5	9.5%	0.5	9.5%	0.5	0.5
2	9.0%	5.1	9.5%	0.5	10.0%	–6.2	–0.2
3	9.0%	0.5	9.75%	–4.3	10.25%	–4.3	–2.7
4	8.75%	3.0	9.5%	3.0	10.5%	–4.3	0.6
5	8.75%	0.5	9.5%	0.5	10.5%	0.5	0.5
6	8.5%	3.0	9.75%	–4.3	10.75%	–4.3	–1.7
7	8.5%	0.5	9.5%	3.0	10.75%	0.5	1.3
8	8.25%	3.0	9.5%	0.5	11.0%	–4.3	–0.8
9	8.25%	0.5	9.75%	–4.3	11.0%	0.5	–1.1
10	8.25%	0.5	9.5%	3.0	11.0%	0.5	1.3

Table 5
PROPOSED PORTFOLIO OF 50% EQUITIES AND 50% FIXED INCOME

Year	Lower Range	Preference	Mid-Range	Preference	Higher Range	Preference	Expected Preference
1	9.5%	0.5	9.5%	0.5	9.5%	0.5	0.5
2	8.75%	7.0	9.5%	0.5	10.0%	–6.2	0.4
3	8.25%	5.1	9.25%	3.0	10.25%	–4.3	1.3
4	7.75%	5.1	9.0%	3.0	10.25%	0.5	2.9
5	7.25%	5.1	8.75%	3.0	10.25%	0.5	2.9
6	7.0%	3.0	8.5%	3.0	10.25%	0.5	2.2
7	7.0%	0.5	8.75%	–4.3	10.5%	–4.3	–2.7
8	6.75%	3.0	8.5%	3.0	10.5%	0.5	2.2
9	6.5%	3.0	8.5%	0.5	10.75%	–4.3	–0.3
10	6.5%	0.5	8.75%	–4.3	10.75%	0.5	–1.1

The contribution rates in the preceding tables are provided for illustrative purposes and cannot be readily duplicated from the information provided. The first year is known to be 9.5%. Future years would depend on the amortization of the change in assumptions, the expected pattern of benefit payments and expected future salaries. Since they are not limited to existing employees, as is the case with actual contribution rate determinations, the projections for future years could vary depending on whether the future salaries and benefits are based on existing employees only or a static or dynamic employee model.

The future preferences were discounted using $\left(\frac{1}{1+|t|}\right)^{k\times|t|}$ with k=0.1 and t = Year – 1, based on the actuary's judgment. The discounted preferences were summed and totaled –1.70 for the current portfolio and 5.73 for the proposed portfolio.

This example is simplified for illustrative purposes and could have also been done using a binary distribution or a stochastic generator and the distribution of outcomes presented to the board for consideration. The trade-off between a lower expected contribution rate and fluctuations in the contribution rates is illustrated here for the board's consideration. A more concise explanation may be to state that due to the annual nature of contribution rate redeterminations and how the assets are valued in determining the contribution rates, the current portfolio strategy is expected to produce contribution rates in the 8.5% to 11.0% range over the next 10 years, with an average contribution rate of about 9.5% to 9.75%. The considered change is expected to result in contribution rates in the 6.5% to 10.75% over the next 10 years with an average contribution rate of about 8.5% to 8.75%.

Given the stated preferences, the actuary could reasonably assume the board of trustees would prefer to adopt the 50% stock and 50% bond portfolio strategy. The key differences in the mean-variance optimization approach and this approach is that descriptive preferences were used to make the determination of which portfolio would be preferred based on the patterns of contribution rates. If the board had not been concerned with the stability of contribution rates or only concerned with contribution rates being at certain levels, the preference function should have been designed to model the board's preferences.

4.3 Preference Functions for a Life Insurance Product Line

As an example of how preference functions may be refined and used in product development we will consider the introduction of a new universal life option. The company currently offers universal life insurance with a choice of options: Option A (i.e., the death benefit is equal to the face amount of the policy) and Option B (i.e., the death benefit is equal to the face amount of the policy plus the account value), with benefits subject to the Cash Value Accumulation Test (CVAT) minimums. Management has expressed a desire for an innovative product for the company's agents to offer potential clients. Specifically, the company's management has been advised by agents that that long-term policyholders have expressed concerns about both Option A and Option B. With Option A the complaint is the death benefit had not kept pace with inflation, and with Option B the complaint is at advanced ages the cost of insurance was so high the death benefit was reduced, sometimes to less than the premiums which had been paid.

Two options are being considered to address the expressed concerns. The first option, referred to as Option C, is a policy that pays a death benefit equal to the face amount plus a return of premiums. The second option, referred to as Option D, is a policy that begins as Option B and between ages 56 and 80 linearly reduces the portion of the account value by 4% per year until the death benefit is the original face amount. Both products have death benefits subject to the CVAT minimums. Management has asked for a recommendation based on a 45-year old male nonsmoker as a representative potential client. To test an actual product, multiple potential clients would likely be considered, and stochastic testing of interest rates and mortality experience may provide more information about the expected preferences and behaviors of the various stakeholders. However, for illustrative purposes we will consider a deterministic scenario with a single client cell; fixed interest rate; fixed mortality assumptions; and a dynamic, deterministic lapse rate assumption.

4.3.1 Modeling Preference Functions for Stakeholders

The actuary considers the stakeholders and their preferences using p_z:Option→[0,1] as the preference function of stakeholder z.

Regulators: For this product the regulators require the product comply with the applicable statutes and that it will not adversely impact the company. The actuary models the preference function as:

$$p_r(Option) = \begin{cases} 1, if\ product\ is\ compliant\ and\ has\ no\ adverse\ impact \\ 0, otherwise. \end{cases}$$

Management: Management's primary concern was that revenues were greater than expenses. They also set an ideal growth rate at 30% with dividends at 10% of capital and surplus. The actuary modeled management's preference function as:

$$p_m(Option) = \left(0.8\ \times \left(1 - \frac{Expenses}{Revenues}\right)\right)$$

$$+0.5\ \times (\min\{0.3, internal\ rate\ of\ return\ after\ taxes - dividends\} + \min\{0.1, \frac{dividends}{.65\ \times capital}\})\ .$$

The company is expected to have a 35% tax rate. As with any of the preference functions used in this model, the actuary could have used a different preference function. Management's preference function is intended to be a model of management's preferences and may be based on the internal rate of return, total profits by year, solvency ratios, or any other metric that describes management's goals and preferences.

Stockholders: The company's stockholders desire return on investment as dividends, as growth in the company or a combination more than the risk-free rate. The model describing their preference used by the actuary was

$$p_s = (0.8 \times dividend\,rate) + growth\,rate — 4\%.$$

Future years were discounted at 8%, the actuary's estimate of the average future long-term return for equities.

The preference was floored at 0 and capped at 1. The growth rate is preferred over dividends because of the tax benefits of capital gains over dividends.

Potential Clients: Potential clients were found to purchase insurance based on their perception of cost and value of the product offered. The equation used to model this by the actuary was

$$p_c = \sqrt{\left(1 - \frac{perceived\ cost}{perceived\ value}\right)}, \qquad \text{floored at } 0.$$

(If 0, the potential clients will not buy the policy and policyholders will lapse an existing policy.) Perceived cost (value) = the discounted value of future premiums (benefits) with a descriptive discount rate based on the discount rate the client would accept in lieu of future payments or the assumed average rate they pay on consumer credit together with mortality based on the client's perceived mortality.

Agents: The agents primary concern was expressed as the amount of commissions they will earn, how soon they will earn them, the ease of selling the product and the amount of service they will be required to provide for a policy. The timing of commissions was the same for both products and the service for the policies was deemed to be proportional to the premium charged and commissions paid. The agents' preference function was modeled as

$$p_a = \frac{average\ commission\ per\ policy}{Max\ of\ average\ commissions\ at\ 100\%\ sales} \times \frac{sales}{presentations}$$

4.3.2 Combining Preference Functions of Stakeholders

If any of the stakeholders has preference of 0 for the new option, it will not be sold. Therefore, the actuary considered a weighted multiple of the preference functions and initially weighted them by the amount of the total premiums expected to go to each:

$$p_{Aggregate}(Option) = p_r^{0.09}(Option)p_m^{0.15}(Option)p_s^{0.03}(Option)p_c^{0.65}(Option)p_a^{0.08}(Option).$$

Since the regulator preference function is binary, it can be removed assuming the product will comply with statutes and management would not approve a product that would be expected to adversely impact solvency. After discussing the initial weightings with management, the actuary decided to simplify the preference function. Since the likelihood of a potential client purchasing the product is included in the agents' preference, it could be removed or embedded in the agents' preference function. Management stressed that it was their responsibility to properly manage the funds for the owners and that the stockholders' preferences were effectively embedded in managements' preference. Management also suggested the 20-year expected income to expense ratio discounted at 12% was adequate. They also suggested adding that the capital required to support the policies based on RBC, which they said could be reasonably estimated using 0.005 times the account value before mortality charges plus 0.0012 times the net amount at risk before mortality charges. They further asserted the regulator portion and part of the client portion should be allocated to management. After these considerations, the actuary modeled the aggregate preference function as:

$$p_{Aggregate}(Option) = \; p_m'^{0.60}(Option)p_a^{0.40}(Option),$$

$$with\; p_m'(Option) = (1 - \frac{PV\ Expenses + PV\ Capital\ Required}{PV\ Revenues}).$$

4.3.3 Assumptions Used in Calculations

The company conducted a study of its potential client base and found the distribution and average premium based on the potential clients perceived mortality and discount rate the potential client considered fair in lieu of a future payment. The actual interest and mortality rates are likely to vary significantly from the illustrated rates. Acceptable interest rates for individuals are indicated by the accepted rates of interest paid on loans and/or the risk-adjusted rates of return expected on investments. Since people tend to overestimate small likelihood of small probabilities, the perception of the mortality rate is likely to be higher than the actual mortality rate. Other factors also will impact an individual's decision of whether to purchase insurance. The rates chosen are intended to illustrate the impact of acceptable discount rates and perceived mortality. The results are summarized in Table 6:

Table 6

DISTRIBUTION OF POTENTIAL CLIENTS AND AVERAGE PREMIUMS PAID IN EACH CLASS

Distribution of Potential Clients & Average Premium		Acceptable Discount Rate for Future Payments			
		2%		8%	
Perceived Mortality	$q^{0.5}$	11%	\$6,000	23%	\$2,500
	$q^{0.8}$	29%	\$12,000	37%	\$4,500

Note: If 100% of the policies were sold, the average premium would be \$6,380.

The reserves are assumed to be the cash surrender values of the policies.

Acquisition costs are 1% of face amount plus \$300.

Commissions are the first year's premium for Option C and 75% of the first year's premium for Option D.

Annual expenses are $150 per policy with 2.5% inflation plus 0.25% of the policy's account value plus 7% of premium. The policy is charged expense charges of 15% of the premium plus a policy fee of $180.

The company's capital requirement for each policy is estimated to be 0.005 times the account value after mortality charges plus 0.012 times the net amount at risk after mortality charges.

Crediting rate is 4.00% and earned rate is 4.75%.

Mortality charges are based on the 2017 CSO standard select and ultimate loaded discounted at 4%. Actual mortality expectations are 2017 CSO standard select and ultimate unloaded.

Lapse rates are estimated by

$$Lapse\ Rate = (1 - client's\ initial\ preference) \times lapse\ duration\ factor.$$

Cash surrender value (CSV) is determined by

$$CSV = Account\ Value\ \times CSV\ duration\ factor.$$

The duration factors are provided in Table 7:

Table 7
LAPSE FACTORS AND CASH SURRENDER VALUE MULTIPLIERS BY DURATION

Duration	Lapse Factor	CSV Factor
1	0.20	0.00
2	0.15	0.60
3	0.10	0.80
4	0.05	0.90
5	0.05	0.95
6	0.15	1.00
7+	0.075	1.00

Premiums and fees are paid at the beginning of the policy year. Benefits are paid at the end of the policy year. For computational simplicity deaths are assumed to occur and then lapses.

For the sample policies, the face amount is set at 20 times the premium less $250 for Option C and 17.5 times the premium less $250 for Option D.

4.3.4 Results for Option C

We will work through the preference for an individual with perceived mortality of $q^{0.5}$ and an acceptable discount rate for future payments of 2% under Option C for the first 5 years. The same approach was used of each combination of acceptable discount rate and perceived mortality. The results are shown in Table 8:

Table 8

DERIVATION OF A CLIENT'S PERCEIVED VALUES FOR OPTION C

Time	Acct Val	Prem. Paid	Expense Charges	Mort. Charge	Interest Credited	Perceived Mortality	Perceived Discount	Death Benefit	Perceived Value of Death Benefits
0	0.00	6,000.00	1,320.00	60.30	194.39	0.019494	0.980392	121,000	2,312.47
1	5,054.09	6,000.00	1,320.00	78.82	395.81	0.022361	0.961169	127,000	2,676.32
2	10,291.08	6,000.00	1,320.00	104.29	604.27	0.025495	0.942322	133,000	3,062.93
3	15,711.06	6,000.00	1,320.00	127.62	820.14	0.028284	0.923845	139,000	3,392.91
4	21,323.58	6,000.00	1,320.00	139.48	1,044.16	0.031464	0.905731	145,000	3,516.25
5	27,148.26	6,000.00	1,320.00	158.03	1,276.41	0.034059	0.887971	151,000	3,716.58

Expanding this to age 121, we can determine the perceived present value of future benefits is \$152,645.41 and the perceived value of a \$6,000 per year immediate annuity is \$87,231.95.

$$p_c = \sqrt{(1 - \frac{perceived\ cost}{perceived\ value})} = \sqrt{(1 - \frac{\$87{,}231.95}{\$152{,}645.41})} = 0.6546$$

The portions of individuals who are modeled to purchase Option C are listed in Table 9.

Table 9

LIKELIHOOD OF PURCHASE OF OPTION C FOR EACH CLASS AND PREMIUM GROUPING

Distribution of Potential Clients & Average Premium		Acceptable Discount Rate for Future Payments	
		2%	8%
Perceived Mortality	$q^{0.5}$	0.6546	0.3199
	$q^{0.8}$	0.4678	0.0000

Using the percentages of the potential clients and expected dollar amount of premiums and the agents' preference function we get

$$p_a(C) = \sum_{all\ cells} \frac{Avg. premium\ for\ cell}{6{,}380} \times probability\ of\ client\ being\ in\ cell \times likelihood\ of\ purchase = 0.3517.$$

Now we will consider the company's income, expenses and capital requirements for the policy with \$6,000 of premium. Results are in Table 10.

Table 10

POLICY CASH FLOWS AND PERSISTENCY FOR A SAMPLE CLIENT WITH OPTION C

Time	Premium	Interest	Reserve Released	Policy Expenses	Death Benefits	Surrender Benefits	Change in Req. Capital	Change in CSV	% of Policies
0	6,000.00	0.00	0.00	8,020.00	0.00	0.00	586.30	0.00	100.0000
1	6,000.00	230.84	0.00	573.75	45.98	0.00	10.80	3,032.45	93.0571
2	6,000.00	470.02	158.54	577.59	63.50	157.02	10.10	5,200.41	88.1920
3	6,000.00	717.57	289.51	581.53	86.45	284.16	9.40	5,907.09	85.0907
4	6,000.00	973.92	255.30	585.57	111.20	243.98	8.67	6,117.45	83.5544
5	6,000.00	1,239.94	367.14	589.71	126.15	349.52	7.87	6,890.86	82.0401

Continuing for 20 years, and discounting using the hurdle rate of 12%, we get the present value of income through year 20 to be \$55,665.32 and the present value of expenses and capital requirements through year 20 to be \$53,073.92.

$$p_m'(Option) = \left(1 - \frac{PV\ Expenses + PV\ Capital\ Required}{PV\ Revenues}\right) = \left(1 - \frac{\$53{,}073.92}{\$55{,}665.32}\right) = 0.0466$$

The preference for the various policy amounts together with their probability of being purchased during a presentation is shown in Table 11.

Table 11

DERIVATION OF MANAGEMENT'S PREFERENCES FOR OPTION C

Policy Amount	Likelihood of Purchase per Presentation	PV 20 Yrs Income	PV 20 Yrs Expenses & Capital	Management's Preference
2,500	0.23 × 0.2567 = 0.059032	18,656.84	18,325.83	0.0177
4,500	0.37 × 0.0000 = 0.000000	0	0	N/A
6,000	0.11 × 0.6546 = 0.072006	55,665.32	53,073.92	0.0466
12,000	0.29 × 0.4816 = 0.139663	102,641.87	98,190.48	0.0434

$$p'_m(C) = 1 - \frac{\sum_{All\ Cells} PV\ Expenses\ \&\ Capital\ for\ cell\ \times probability\ of\ purchase}{\sum_{All\ Cells} PV\ Income\ for\ cell\ \times probability\ of\ purchase} = 1 - \frac{\$18{,}617.03}{\$19{,}444.86}$$

$$= 0.0426.$$

Thus,

$$p_{Aggregate}(C) = p_m'^{0.60}(C)p_a^{0.40}(C) = 0.0426^{0.6} \times 0.3518^{0.4} = 0.0991$$

4.3.5 Results for Option D

We will work through the preference for an individual with perceived mortality of $q^{0.8}$ and an acceptable discount rate for future payments of 8% under Option D for the first 5 years. The same approach was used of each combination of acceptable discount rate and perceived mortality. The annual premium is $4,500 and the face amount is $74,375. Results are presented in Table 12.

Table 12
DERIVATION OF A CLIENT'S PERCEIVED VALUES FOR OPTION D

Time	Acct Val	Prem. Paid	Expense Charges	Mort. Charge	Interest Credited	Perceived Mortality	Perceived Discount	Death Benefit	Perceived Value of Death Benefits
0	0.00	4,500.00	855.00	38.71	144.25	0.001836	0.925926	78,165.80	132.87
1	3,750.54	4,500.00	855.00	50.29	293.81	0.002287	0.857339	82,066.36	160.58
2	7,639.06	4,500.00	855.00	66.25	448.71	0.002821	0.793832	86,110.42	192.01
3	11,666.52	4,500.00	855.00	80.85	609.23	0.003330	0.735030	90,298.98	219.50
4	15,839.90	4,500.00	855.00	88.27	775.87	0.003561	0.680583	94,639.30	227.04
5	20,172.50	4,500.00	855.00	100.09	948.70	0.003949	0.630170	99,145.20	243.34

Expanding this to age 121, we can determine the perceived present value of future benefits is $21,673.31 and the perceived value of a $4,500 per year immediate annuity is $53,630.38. Since the perceived cost is greater than the perceived value, the client would not purchase the policy and the preference is 0.

The portions of individuals who are modeled to purchase Option D are listed in Table 13.

Table 13
LIKELIHOOD OF PURCHASE OF OPTION D FOR EACH CLASS AND PREMIUM GROUPING

Distribution of Potential Clients & Average Premium		Acceptable Discount Rate for Future Payments	
		2%	**8%**
Perceived Mortality	**$q^{0.5}$**	0.6030	0.0000
	$q^{0.8}$	0.4696	0.0000

Using the percentages of the potential clients and expected dollar amount of premiums and the agents' preference function we get

$$p_a(D) = \sum_{all\ cells} \frac{75\%\ of\ Avg.premium\ for\ cell}{\$6{,}380} \times probability\ of\ client\ being\ in\ cell \times likelihood\ of\ purchase$$

$$= 0.2389.$$

Looking at Table 14, now we will consider the company's income, expenses and capital requirements for the policy with annual premium of $12,000 and $205,625 in face amount of insurance.

Table 14
POLICY CASH FLOWS AND PERSISTENCY FOR A SAMPLE CLIENT WITH OPTION D

Time	Premium	Interest	Reserve Released	Policy Expenses	Death Benefits	Surrender Benefits	Change in Req. Capital	Change in CSV	% of Policies
0	12000.00	0.00	0.00	15346.25	0.00	0.00	1042.15	0.00	100.0000
1	12000.00	470.87	0.00	993.75	82.10	0.00	14.44	6185.69	89.3580
2	12000.00	959.05	494.98	997.59	113.38	491.89	14.96	10612.75	82.2076
3	12000.00	1464.66	901.33	1001.53	154.62	890.41	15.50	12063.01	77.7967
4	12000.00	1988.58	787.88	1005.57	199.52	764.80	16.06	12500.94	75.6729
5	12000.00	2532.47	1131.96	1009.71	227.35	1095.98	16.67	14085.56	73.6020

Continuing for 20 years, and discounting using the hurdle rate of 12%, we get the present value of income through year 20 to be \$102,159.83 and the present value of expenses and capital requirements through year 20 to be \$97,374.84.

$$p'_m(Option) = \left(1 - \frac{PV\ Expenses + PV\ Capital\ Required}{PV\ Revenues}\right) = \left(1 - \frac{\$97{,}374.84}{\$102{,}159.83}\right) = 0.0468$$

The preference for the various policy amounts together with their probability of being purchased during a presentation are provided in Table 15.

Table 15
DERIVATION OF MANAGEMENT'S PREFERENCES FOR OPTION D

Policy Amount	Likelihood of Purchase/Presentation	PV Income	PV Expenses & Capital	Management's Preference
2,500	0.23 × 0.0000 = 0.000000	0.00	0.00	N/A
4,500	0.37 × 0.0000 = 0.000000	0.00	0.00	N/A
6,000	0.11 × 0.6030 = 0.066330	54,340.78	51,734.39	0.0480
12,000	0.29 × 0.4696 = 0.136184	102,159.83	97,374.84	0.0468

$$p'_m(D) = 1 - \frac{\sum_{All\ Cells} PV\ Expenses\ \&\ Capital\ for\ cell\ \times probability\ of\ purchase}{\sum_{All\ Cells} PV\ Income\ for\ cell\ \times probability\ of\ purchase} = 1 - \frac{\$16{,}692{,}44}{\$17{,}516.96}$$
$$= 0.0471.$$

Thus,

$$p_{Aggregate}(D) = p_m'^{0.60}(D) p_a^{0.40}(D) = 0.0471^{0.6} \times 0.2389^{0.4} = 0.0902.$$

4.3.6 Concluding Results

Management has a higher preference for Option D (0.0471 vs. 0.0426), and agents have a higher preference for Option C (0.3517 vs 0.2389). The aggregated preference function indicates the highest level of satisfaction would be obtained with Option C (0.0991 vs 0.0902).

For illustrative purposes, the input data in this example were manipulated to produce results with agents and management having different preferences. An alternative approach is to remove the preferences of the agents by increasing the percentage of first year's premium paid as a commission on Option D to the level that the agents have no preference between the two options, and then determine which option management prefers.

Section 5: Conclusions and Further Developments

Behavioral psychology plays a crucial role in human preferences and decision-making processes. The use of descriptive preference functions can improve the accuracy and predictive value of a model when compared to normative utility functions, especially when the behavior or reality being modeled does not conform with the restrictions of a utility function. The differences in economic utility and psychological preferences create opportunities for profits and loss avoidance. The use of actuarial preference functions could benefit the areas of investments, product development, marketing, loss mitigation, pricing, pensions and predictive analytics.

Although we have considered preference function as a deterministic function, which describes the choices of a decision maker, it could be further generalized into a stochastic preference function that provides the likelihood or distribution of choices in the population being considered. For example, such a function could provide the percentages of the population who would purchase various amounts of insurance, given the options available to them. This will lend itself to potential advances in stochastic modeling. There is opportunity to consider deterministic preference functions on a set of finite choices through applications from graph theory related to complete directed graphs.

Prospect Theory and preference functions are not yet well developed within the actuarial practice areas. Given that Prospect Theory is driving innovations in economics, medicine and political science, it is reasonably likely that actuarial models will need to include preference functions to keep abreast of other sciences and professions. In some areas of actuarial practice, limited preference functions are already being employed. For example, preference functions often are used to model dynamic lapse rates or increases in retirement rates when individuals become eligible for certain levels of social security benefits. However, there are many other opportunities for actuaries to employ descriptive preference functions and prospect theory in their models to better reflect actual human behavior. A well-designed, descriptive preference function would be expected to provide a reflection of realistic behaviors based on preferences between choices as part of the model and not to rely solely on the economic values of states. Including descriptive preference functions as a model for human behaviors in actuarial models can improve the accuracy of the model results, improve the quality of analysis used for decision making, improve product design and identify potentially otherwise overlooked areas where changes may be in order.

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