Aging and Retirement

Exploration of Lifetime Pension Pool Design Elements

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Executive Summary

Several modern retirement arrangements, including lifetime pension pools, allow retirees to convert a single premium into income for life that varies with investment and mortality experience. It is anticipated that there will be an increase in the number of lifetime pension pools in the future. There are, nonetheless, some impediments to the proliferation of lifetime pension pools: the regulatory framework in North America, which is not fully ready for these pools; the lack of efficient and reliable ways to communicate and disclose risk to members; and no consensus yet on optimal pool design elements.

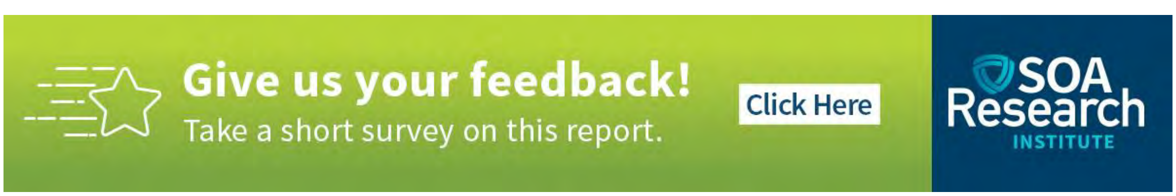
The focus of the present report is the third challenge above; that is, pool design. We explore three different elements:

1. Closed and open pools: Many studies in the literature featured pools that do not allow members to join after inception (i.e., closed pools). These pools are easier to manage because you do not need to account for any new members joining the pool when calculating the experience adjustment factors. Yet allowing new members to join the pool after its inception (i.e., open pools) increases the potential for keeping the number of pool members at a decent level, thus increasing the robustness of mortality pooling by keeping the mortality adjustment uncertainty at a reasonable level. This assumes, however, that prospective members are willing to join the pool and that the pool can attract new members.
2. Hurdle rate policy: So far, most of the literature has focused on pools with a constant hurdle rate set at the pool's inception. It would, nonetheless, be possible to consider a variable hurdle rate in practice, which could be linked to return expectations. If done correctly, hurdle rate adjustments may reduce year-to-year volatility in benefits and adjustments. Additionally, not adjusting the hurdle rate when return expectations change may diminish fairness, especially in the context of open pools. On the other hand, a variable hurdle rate policy might prove to be a challenge from a communication perspective for pool operators: it might be difficult for some members to understand the nature of changing (hurdle) rates in this context.
3. Delayed recognition of gains and losses: Delayed recognition could give participants a longer time to adjust their consumption to changes in pension income. However, it can introduce value transfers among members if done poorly. Moreover, it can also create the potential for shortfall (the pool having fewer assets than liabilities) and bankruptcy (the pool not having assets anymore). Some empirically relevant methods for delayed recognition are the staggering of benefit changes over consecutive years and the creation of an adjustment corridor within which the benefits are not changed. In this report, and in addition to the two procedures above, we also consider a method based on adjusting the hurdle rate.

The report shows that most design elements give rise to trade-offs between risk and reward. For instance, open pools lead to significantly less risk than closed pools, especially once closed pools consist primarily of older members. Yet open pools yield average benefits that are slightly lower than those of closed pools with older members. This is thanks to the mortality credit asymmetry in lifetime pension pools, which occurs when the number of members in the pool decreases. Hybrid measures that take both risk and reward into account identify the open pool's performance as superior over longer horizons.

When compared to a fixed hurdle rate, a variable rate reduces the standard deviation of benefit adjustments but increases the standard deviation of benefits. It is unclear if members care more about volatility in the benefits or their adjustments, so pool designers should be vigilant when selecting their hurdle rate assumptions.

Finally, delayed recognition of gains and losses can significantly reduce the risk associated with changing benefits. These methods, however, tend to increase the risk of shortfall and bankruptcy, which could impact fairness and the ability to attract new members to the pool.



Section 1: Introduction, Literature Review, and Scope

With the decrease in the prevalence of guaranteed pension arrangements worldwide, flexible schemes such as lifetime pension pools are expected to gain in popularity. These pools enable retiring individuals to convert a lump sum into a lifelong income. However, lifetime pension pools do not guarantee a fixed income level, unlike traditional pension arrangements and annuities. Instead, the amount of pension payable varies based on the group's investment and mortality experience.

It has been suggested in the literature that pooling individual participants' mortality risks has the potential to generate higher sustainable income while members are alive when compared to individual systematic withdrawal plans (see, e.g., Australian Government Actuary, 2014). Lifetime pension pools are also expected to outperform insured annuities from a utility standpoint, as there is no requirement for risk capital to support guarantees (see, e.g., Chen and Rach, 2023).

Various arrangements, products, and monikers fit the broad description of lifetime pension pools in the literature: group self-annuitization plans (Piggott et al., 2005; Valdez et al., 2006; Qiao and Sherris, 2013; Hanewald et al., 2013), pooled annuity funds (Stamos, 2008; Sabin, 2010; Donnelly et al., 2013), annuity overlay funds (Donnelly et al., 2014; Donnelly, 2015), retirement tontines (Milevsky and Salisbury, 2015, 2016; Sabin and Forman, 2016; Fullmer and Sabin, 2019a, 2019b; Fullmer, 2019; Iwry et al., 2020; Chen et al., 2021; Winter and Planchet, 2022), assurance funds (Fullmer and Forman, 2022), variable annuities (Balter and Werker, 2020; Balter et al., 2020), variable payout annuities (Horneff et al., 2010; Boyle et al., 2015), dynamic pension pools (MacDonald et al. 2021), and variable payment life annuities (Association of Canadian Pension Management, 2017).[1]

Several examples of lifetime pension pools exist in practice, such as the College Retirement Equities Fund, which has been operated by the Teachers Insurance and Annuity Association of America (TIAA) in the United States since 1952. The variable payment life annuities offered by the Faculty Pension Plan of the University of British Columbia (UBC) since 1967 represent another example. More recently, QSuper (now part of the Australian Retirement Trust) introduced the Lifetime Pension to the Australian market in 2021. Two Canadian financial services firms followed suit in 2021 and 2022, respectively: the Longevity Pension Fund organized and managed by Purpose Investments, and the GuardPath Modern Tontine Trust by Guardian Capital.

It is anticipated that there will be an increase in the number of lifetime pension pools in the future. There are, however, some impediments to their widespread adoption:

1. Regulatory framework: The current regulatory framework in North America is not ready for the proliferation of lifetime pension pools. Recent changes to income tax regulations accompanying the Budget Implementation Act of 2021 in Canada are expected to help; however, more work still needs to be done at the provincial level. In the United States, we recognize that establishing such a pool within the current legal framework might pose difficulties for private pension plan sponsors given the constraints imposed by ERISA and federal securities regulation laws. However, it is worth noting that public sector and religious organizations are largely exempt from the majority of ERISA's pension regulations, allowing these employers to theoretically implement such pools without violating federal law (Forman and Sabin, 2015). In addition, there is some interest among pension practitioners, as seen in Shemtob (2021, 2022).

[1] In recent years, there has been a great deal of academic research on lifetime pension pools and tontine-like designs, which has led to the introduction of numerous new products and designs (see, e.g., Chen and Rach, 2019; Chen et al., 2020). For example, some of the new products include combinations of annuities and tontines, like tonuities (Chen et al., 2019), tontines with bequest (Bernhardt and Donnelly, 2019), and unit-linked tontines (Chen et al., 2022), among others.

2. Communication and disclosure: Given that the income provided through lifetime pension pools is expected to vary as a function of the investment and mortality experience, it is paramount to characterize the associated benefit risk and be able to communicate it to pool members. Indeed, actuaries and plan sponsors need to communicate risk to members meaningfully so that members can make appropriate decisions about retirement income. Recent work by Bégin and Sanders (2023) provided some insights on how to communicate and disclose risk to members in the context of lifetime pension pools; their collection of measures—called *benefit at risk* or *BaR*—can inform members of the extent of possible benefit losses compared to a specific benchmark over a particular time frame. Yet there is more work to be done on this front to ensure members get adequate information about the risk they bear.
3. Optimal pool design elements: The design of a lifetime pension pool affects the pool's risk–reward profile, which can significantly impact members' benefits during retirement. Some research has been done on these design elements, but many questions remain in this domain. For instance, is it better to allow members to join the pool after its inception or only when it is created? What hurdle rate should be used to compute the adjustment factors? Should the recognition of gains and losses be delayed somehow? If so, how?

The purpose of this report is to explore the last challenge—optimal design elements. We do not aim to provide clear guidance on exact design choices, but rather to provide a starting point for a dialogue between actuaries, pool operators, regulators, financial advisors, academics, and potential lifetime pension pool members.

We specifically focus on three different dimensions of design:

1. Closed and open pools: Many studies in the literature featured pools that do not allow members to join after inception (i.e., closed pools). These pools are easier to manage because they do not need to account for any new members when calculating the experience adjustment factors. Yet allowing members to join the pool after its inception (i.e., open pools) can increase the robustness of mortality pooling by keeping the mortality adjustment uncertainty at a reasonable level. This assumes, however, that prospective members are willing to join the pool.[2]
2. Hurdle rate policy: So far, most of the literature has focused on pools with a constant hurdle rate set at the pool's inception. It would, nonetheless, be possible to consider a variable hurdle rate in practice, which could be linked to return expectations. On the one hand, hurdle rate adjustments may reduce year-to-year volatility in benefits and adjustments if done correctly. Additionally, not adjusting the hurdle rate when return expectations change may diminish fairness, especially in the context of open pools. On the other hand, a variable hurdle rate policy might prove to be a challenge from a communication perspective for pool operators: it might be difficult for some members to understand the nature of changing (hurdle) rates in this context.
3. Delayed recognition of gains and losses: Delayed recognition could give participants a longer time to adjust their consumption to changes in pension income. However, it can introduce value transfers among members if done poorly. Moreover, it can also create the potential for shortfall (or deficit) and bankruptcy of the pool. In this report, we consider three methods for delayed recognition: (a) staggering benefit changes over consecutive years, (b) creating an adjustment corridor within which the benefits are not changed, and (c) adjusting the hurdle rate in a manner inspired by the Royal Mail Collective Defined Contribution pension plan in the United Kingdom.

[2] Bernhardt and Donnelly (2021) investigated the question of membership size and income stability in a recent paper. They also provided guidance in terms of pool size. The question of pool size is related to our concern about closed and open pools, as open pools allow for the membership to grow, whereas closed pools' memberships will only decrease over time.

To assess these different design elements, we consider many different risk and reward measures.[3] To evaluate potential rewards, we rely on the average benefits over different relevant periods for pensioners. To assess risk, we use the standard deviation of adjustments and of benefits. These measures are supplemented by the shortfall probability of adjustments (i.e., the probability of observing adjustments below a given threshold) and the shortfall probability of benefits (i.e., the probability of observing benefits below a given threshold).

When considered separately, these risk and reward measures only tell one part of the story, so we also look at three *hybrid* measures that capture various combinations of risk and reward. First, we consider the minimum BaR recently introduced by Bégin and Sanders (2023). This measure is based on the minimum benefit likely to be realized over a relatively short period; it tends to be helpful as a budgeting tool (i.e., for existing participants who must plan for and potentially adjust consumption based on the income provided by the pool). Second, we use the average benefit at risk, also presented in Bégin and Sanders (2023). The latter measure evaluates the long-term risk and reward by comparing the expected average benefits to potential benefits in extreme scenarios over a longer horizon—in other words, the benefit shortfall. Third, we rely on a utility-based measure to combine the various benefit streams and their uncertainty. Specifically, we use certainty equivalent consumption, which captures members' preferences.

This report is structured as follows. Section 2 introduces two stylized versions of lifetime pension pools for illustrative purposes: the first version relies on group-based adjustments in the spirit of Piggott et al. (2005) and Qiao and Sherris (2013), whereas the second version is based on cohort adjustments similar to the update rules recently put forward by Fullmer and Sabin (2019a) and Fullmer (2019) for modern tontines. Section 3 presents various reward, risk, and hybrid measures to assess these dimensions in the context of lifetime pension pools. Open and closed pools are compared in Section 4. Section 5 reports results on the hurdle rate policy. Delays in recognition of gains and losses are investigated in Section 6. Section 7 concludes and discusses avenues for future research. Two appendices explain in greater detail the economic scenario generator (Appendix A) and the mortality model (Appendix B) used in the report.

[3] We acknowledge that some of our design choices and assumptions are subjective. We encourage readers to compare their own designs using the various measures proposed in this report.

Section 2: An Introduction to Benefit Update Rules in Lifetime Pension Pools

As mentioned in the introduction, some lifetime pension pool designs already exist. This section first provides a general framework for benefit updates by building on the current literature on such pools.

We then give two specific examples considering group-based adjustments and cohort-specific adjustments, respectively. The former rule is inspired by Piggott et al. (2005), Valdez et al. (2006), Qiao and Sherris (2013), and Hanewald et al. (2013) in the context of group self-annuitization plans. It is also reminiscent of the benefit update rule used by the College Retirement Equities Fund (CREF, 2022a,b) and UBC's Faculty Pension Plan. The latter adjustment rule is similar to that used in tontines and tontine-like designs. This report follows the fair individual tontine update rule put forward by Fullmer and Sabin (2019a) and Fullmer (2019).

This section concludes with a simple numerical example comparing both schemes.

2.1 GENERAL DYNAMICS FOR VARYING ANNUITY PAYOUTS

We begin this section by introducing a generic framework for varying annuity payouts that embeds many specific benefit update rules in the literature. The goal of this section is not to provide an exhaustive review of all possible update rules but rather to explain two practical examples that are adopted later in this report.

We use $\mathcal{L}_t$ to represent the set of survivors at time t; that is, $k \in \mathcal{L}_t$ if and only if the k^{th} member is alive at time t. We consider the potential for open pools in this report; to account for the possibility of including new members, we define the set $\mathcal{L}_{t^+}$ representing the members of the pool at time t after new members have joined. For convenience, we also define a set of decedents between time $t-1$ and t, which can be obtained from the sets of survivors:

$$\mathcal{D}_{t-1} = \mathcal{L}_{t-1^+} \cap \mathcal{L}_t^{\mathrm{c}},$$

where $\mathcal{L}_t^{\mathrm{c}}$ is the complement of set $\mathcal{L}_t$ (i.e., all deceased members at time t since the pool's inception). In other words, the set of decedents is the intersection between all deceased members at time t since inception and those who survived up to $t-1$.

Note that, for convenience's sake, we assume that the stylized lifetime pension pool pays benefits at the beginning of the year (and that time is given in years). Let $B_k(t)$ be the time-t benefit for the k^{th} life. We assume in all generality that the time-t benefit of this member is updated by using the following rule:

$$B_k(t) = \alpha_k(t) \times B_k(t-1), \tag{1}$$

where $\alpha_k(t)$ is the time-t adjustment factor informed by the experience of the relevant group (pool, cohort, etc.) as well as any other design features such as changes in the hurdle rate policy and delayed recognition of gains and losses (see Sections 5 and 6, respectively, for more details). Simply put, the adjustment factor captures the changes from one year's benefit to the next.

Note that the adjustment factor can be member specific in Equation (1), hence explaining its dependence on the k^{th} member; indeed, we allow for the potential of having different updates for different members in the plan, in line with their specific risk (e.g., age and sex).

Further, we assume that this adjustment is constructed from changes relating to mortality experience and investment experience such that

$$\alpha_k(t) = \mathrm{MEA}_k(t) \times \mathrm{IEA}_k(t).$$

This formula implies that the adjustment applied at time t is the product of the time-t mortality experience adjustment factor, $\mathrm{MEA}_k(t)$, and the time-t investment experience adjustment, $\mathrm{IEA}_k(t)$.[4,5] Again, these mortality and investment experience adjustment factors are—or at least could be—member specific.[6,7] We apply these update rules until no members are left in the pool.[8]

In the following sections, we consider two specific benefit update rules: the first rule assumes that the benefit adjustment factor is the same for every pool member, and the second uses a rule that is specific to each cohort; that is, members of the same sex and age. These two methods are not exhaustive; many others exist. We use them for convenience and because they have been used in practice thus far. Our selection of these illustrative benefit update rules should not be interpreted as guidance on the optimality or desirability of these specific rules. Readers already familiar with these methods who wish to focus on design-related results might skip Sections 2.2, 2.3, and 2.4.

2.2 GROUP-BASED ADJUSTMENTS

The operation of the pool with group-based adjustments is similar to those explained in Piggott et al. (2005) and Qiao and Sherris (2013) in the context of group self-annuitization plans; it is also the update rule used in the stylized pool investigated in Bégin and Sanders (2023). It describes how the amount left by decedents is shared among surviving members, among others.

Let $\ddot{a}^{[s]}_{x_k,t}$ denote the price of an annuity due at time t for the k^{th} life, who is assumed to be aged x_k at inception (i.e., time 0); the time at which the price is determined is denoted in the subscript. The superscript—in brackets—refers to the assumptions used in the valuation; $[s]$ means that we use the mortality table and the interest rate structure available at time s to obtain the annuity price. The time in the subscript and that in the superscript could differ.[9]

Specifically, in this report, the annuity prices are obtained based on two main assumptions:

1. A generational life table that is representative of systematic mortality in the pool and that accounts for mortality improvements is used. This table could be updated occasionally, and our notation allows for this (see Appendix B for more details on the mortality assumptions used in this report).
2. A term structure of interest rates is used to discount cash flows. If the term structure used for discounting is flat, we denote the constant (continuously compounded) interest rate applicable at time t by $h^{[t]}$. This is the so-called *hurdle rate*.[10,11]

[4] For a simple numerical example of benefit updates in the context of lifetime pension pools, see Section 3.2 of Bégin and Sanders (2023).

[5] We only consider investment experience adjustment factors that are shared among all members. Designs with cohort-specific investment experience adjustments also exist; these can redistribute actual investment gains and losses among cohorts to, for example, stabilize benefit streams at older ages.

[6] We follow the literature and exclude data corrections, expenses, administrative fees, and amortization of set-up costs from our modelling because these additional elements would only have second-order effects on our end results.

[7] It is common in the lifetime pension pool literature to separate the effects of investment and mortality (see, e.g., Piggott et al., 2005; Qiao and Sherris, 2013; Boyle et al., 2015; Bégin and Sanders, 2023). Note that the adjustment does not need to be set up in this way; for instance, both mortality and investment adjustments can be combined (see, e.g., Eadie, 2017).

[8] There are many ways to handle the last survivor in a closed pool. For instance, the last survivor could be refunded the remaining balance of the pool, or the pool could buy an insured annuity to the last member. Alternatively, the pool could continue paying the last survivor using the same rules—this is our approach.

[9] If the mortality table and the interest rate assumptions are not updated between time s and $s+1$, then $\ddot{a}^{[s]}_{x,t} = \ddot{a}^{[s+1]}_{x,t}$.

[10] If the term structure is not flat, the rate used to discount a specific cash flow depends on both the time of valuation and the time at which the cash flow is paid. In this case, the concept of a single hurdle rate is not meaningful; instead, the full term structure needs to be taken into account when computing the annuity price.

[11] We use continuously compounded rates for mathematical convenience in this report. We acknowledge that some practitioners might be more familiar with effective annual rates. In this case, one can simply replace the discounting factors. These substitutions should not change the results presented in this report, assuming that continuously compounded rates are appropriately converted into effective annual rates.

When the term structure used for discounting is flat, the following recursive relationship holds:

$$\ddot{a}_{x_j,t-1}^{[t-1]} - 1 = \; p_{x_k+t-1}^{[t-1]} \, \ddot{a}_{x_j,t}^{[t-1]} \; \exp\left(-h^{[t-1]}\right). \tag{2}$$

Assume that at time $t-1$, each member k of the pool has an investment amount of $A_k(t-1)$ so that the total asset value is given by[12]

$$A(t-1) = \sum_{k\in\mathcal{L}_{t-1^+}} A_k(t-1).$$

For each member, the benefit $B_k(t-1)$ at time $t-1$ can be obtained from

$$B_k(t-1) = \frac{A_k(t-1)}{\ddot{a}_{x_k,t-1}^{[t-1]}}. \tag{3}$$

At time t, the total asset value becomes

$$A(t) = \left(A(t-1) - \sum_{j\,\in\,\mathcal{L}_{t-1^+}} B_j(t-1) \right) \exp(r_t^{\mathrm{PF}}) = \sum_{j\,\in\,\mathcal{L}_{t-1^+}} B_j(t-1) \left(\ddot{a}_{x_j,t-1}^{[t-1]} - 1 \right) \exp(r_t^{\mathrm{PF}}),$$

where r_t^{PF} is the time-t continuously compounded rate of return realized on the asset portfolio.[13] The equation above is obtained retrospectively: it is a roll-forward of assets using actual benefit payments and actual investment returns. One can also get a value for $A(t)$ in a prospective fashion from Equation (1):

$$A(t) = \sum_{j\,\in\,\mathcal{L}_t} B_j(t)\,\ddot{a}_{x_j,t}^{[t]} = \sum_{j\,\in\,\mathcal{L}_t} \left(\alpha_j(t)\, B_j(t-1) \right) \ddot{a}_{x_j,t}^{[t]}.$$

Simply put, the value of the adjusted future benefits should equal the accumulated value of the assets. Equating the retrospective and prospective values for $A(t)$ and assuming the same adjustment is applied to all surviving members' benefits at time t gives

$$\alpha_k(t) = \alpha(t) = \left(\frac{\sum_{j\,\in\,\mathcal{L}_{t-1^+}} B_j(t-1) \left(\ddot{a}_{x_j,t-1}^{[t-1]} - 1 \right)}{\sum_{j\,\in\,\mathcal{L}_t} B_j(t-1)\,\ddot{a}_{x_j,t}^{[t]}} \right) \exp(r_t^{\mathrm{PF}}).$$

Applying the recursion from Equation (2) to the annuity factors yields the following expression:

$$\begin{aligned} \alpha(t) \quad &= \left(\frac{\sum_{j\,\in\,\mathcal{L}_{t-1^+}} B_j(t-1)\, p_{x_j+t-1}^{[t-1]}\, \ddot{a}_{x_j,t}^{[t-1]}}{\sum_{j\,\in\,\mathcal{L}_t} B_j(t-1)\,\ddot{a}_{x_j,t}^{[t]}} \right) \times \exp\left(r_t^{\mathrm{PF}} - h^{[t-1]}\right) \\ &= \; \mathrm{MEA}(t) \times \mathrm{IEA}(t). \end{aligned} \tag{4}$$

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[12] These are notional amounts used only to calculate the benefit adjustments. Members do not have a direct entitlement to this amount: upon a member's death the entire amount is forfeited and redistributed to survivors.

[13] The factor $\exp(r_t^{\mathrm{PF}})$ could be replaced with $(1 + \tilde{r}_t^{\mathrm{PF}})$, where $\tilde{r}_t^{\mathrm{PF}} = \exp(r_t^{\mathrm{PF}}) - 1$ is the effective annual rate equivalent to the continuously compounded rate r_t^{PF}.

This version of the mortality experience adjustment contains sums of annuity factors under the *old* mortality and interest rate assumptions (i.e., those applicable at time $t-1$) in the numerator and under the *new* mortality assumptions and interest rate assumptions (i.e., at time t) in the denominator, weighted by the number of retirees at each age and the size of their benefits, thus accounting for both experience arising from actual versus expected deaths and the impact of changes in the actuary's underlying assumptions. If we keep the same (generational) mortality table and the same hurdle rate structure between time $t-1$ and t, we have $\ddot{a}_{x_j,t}^{[t]} = \ddot{a}_{x_j,t}^{[t-1]}$.

Note that these mortality and investment adjustment factors are the same for each member; that is, they do not depend on k. Ultimately, this means that every member's benefit changes by the same proportion, regardless of their sex, age, and the amount of investment they brought to the pool when they joined.

2.3 COHORT-SPECIFIC ADJUSTMENTS

We now focus on a slightly different way of allocating the decedents' account values (i.e., their forfeited balances) among the survivors, leading to a cohort-specific update rule. It is inspired by the literature on fair tontines and tontine-like designs, where member-specific characteristics are used to determine how the benefits are updated, thus leading to different mortality adjustments for each cohort (see, e.g., Fullmer and Sabin, 2019a; Fullmer, 2019). This method is known as the nominal-gain method by Sabin and Forman (2016).

For an investment to be fair, the expected value of the gains and losses must be zero for each member; this equality is known as the fairness constraint in this literature. In our context, this means that the expected value of each pool member's total return on their investment needs to be the same as the total return the member would have obtained outside the pool. Assuming that the k^{th} member has an investment amount of $A_k(t-1)$ at time $t-1$, the fairness constraint implies that

$$\mathbb{E}_{t-1}\left[\left(A_k(t-1) - B_k(t-1)\right)\exp(r_t^{\text{PF}})\right] = q_{x_k+t-1}^{[t-1]} \times 0 + \left(1 - q_{x_k+t-1}^{[t-1]}\right) \times \mathbb{E}_{t-1}[\, A_k(t) \mid \text{Member } k \text{ survives}\,],$$

where $\mathbb{E}_{t-1}[\, A_k(t) \mid \text{Member } k \text{ survives}\,]$ is the expected value of the member's account value conditional on their survival and the information acquired up to time $t-1$. Solving for the conditional expectation gives us

$$\begin{aligned}\mathbb{E}_{t-1}[\, A_k(t) \mid \text{Member } k \text{ survives}\,] &= \frac{\mathbb{E}_{t-1}\left[\left(A_k(t-1) - B_k(t-1)\right)\exp(r_t^{\text{PF}})\right]}{1 - q_{x_k+t-1}^{[t-1]}} \\ &= \mathbb{E}_{t-1}\left[B_k(t-1)\left(\ddot{a}_{x_k,t-1}^{[t-1]} - 1\right)\exp(r_t^{\text{PF}})\right] \times \left(1 + y_k(t)\right),\end{aligned}$$

where

$$y_k(t) = \frac{q_{x_k+t-1}^{[t-1]}}{1 - q_{x_k+t-1}^{[t-1]}}$$

is the member's so-called *nominal yield in excess of the investment returns* (*nominal yield* henceforth). This yield increases the benefits of all survivors by reallocating the expected forfeitures of decedents in relation to each member's mortality risk: it implies higher adjustments for older members with larger death probabilities and lower adjustments for younger members with smaller death probabilities.

In addition to this nominal yield, which accounts for expected mortality outcomes, the benefits should also be adjusted to account for deviations in the pool's actual experience from expected—called the *group gain* in this literature. This group gain is found by dividing the sum of balances forfeited by those who died during the year by the sum of the nominal gains of those who survived:

$$G(t)=\frac{\sum_{j\,\in\,\mathcal{D}_{t-1}}\left(A_j(t-1)-B_j(t-1)\right)\,\exp(r_t^{\mathrm{PF}})}{\sum_{j\,\in\,\mathcal{L}_t}\,y_j(t)\,\left(A_j(t-1)-B_j(t-1)\right)\,\exp(r_t^{\mathrm{PF}})}=\frac{\sum_{j\,\in\,\mathcal{D}_{t-1}}\,B_j(t-1)\left(\ddot{a}_{x_j,t-1}^{[t-1]}-1\right)}{\sum_{j\,\in\,\mathcal{L}_t}\,q_{x_j+t-1}^{[t-1]}\,B_j(t-1)\,\ddot{a}_{x_j,t}^{[t-1]}\,\exp\left(-h^{[t-1]}\right)}$$

according to Equation (2). If the actual mortality experience exactly matches expectations and the valuation basis does not change, the group gain is one.

Once we know the nominal yield and the group gain, we can find each member's account value by combining both components with the time-t investment returns,

$$\begin{aligned}A_k(t)&=\left(A_k(t-1)-B_k(t-1)\right)\,\exp(r_t^{\mathrm{PF}})+\left(A_k(t-1)-B_k(t-1)\right)\,\exp(r_t^{\mathrm{PF}})\,\,y_k(t)\,G(t)\\&=B_k(t-1)\left(\ddot{a}_{x_k+t-1}^{[t-1]}-1\right)\,\exp(r_t^{\mathrm{PF}})\left(1+\,y_k(t)\,G(t)\right),\end{aligned}$$

simply by adding the k^{th} member's share of the decedents' forfeiture.

At the pool level, this allocation is complete. The total accumulated value of members' accounts from the prior period is equal to the sum of the account values of current survivors:

$$\sum_{j\,\in\,\mathcal{L}_{t-1^+}}\left(A_j(t-1)-B_j(t-1)\right)\,\exp(r_t^{\mathrm{PF}})=\sum_{j\,\in\,\mathcal{L}_t}A_j(t).$$

The equation above is obtained retrospectively. One can also get a value for $A_k(t)$ in a prospective fashion:

$$A_k(t)=B_k(t)\,\ddot{a}_{x_k,t}^{[t]}=\alpha_k(t)\,B_k(t-1)\,\ddot{a}_{x_k,t}^{[t]}. \tag{5}$$

Equating the retrospective and prospective values for $A_k(t)$ gives

$$\begin{aligned}\alpha_k(t)&=\frac{B_k(t-1)\,\left(\ddot{a}_{x_k,t-1}^{[t-1]}-1\right)\,\exp(r_t^{\mathrm{PF}})\left(1+y_k(t)\,G(t)\right)}{B_k(t-1)\,\ddot{a}_{x_k,t}^{[t]}}\\&=p_{x_k+t-1}^{[t-1]}\,\frac{\ddot{a}_{x_k,t}^{[t-1]}}{\ddot{a}_{x_k,t}^{[t]}}\,\left(1+y_k(t)\,G(t)\right)\exp\left(r_t^{\mathrm{PF}}-h^{[t-1]}\right)\end{aligned}$$

by again using Equation (2). We can further simplify the equation above by using the definition of the nominal yield:

$$\begin{aligned}\alpha_k(t)&=\frac{\ddot{a}_{x_k,t}^{[t-1]}}{\ddot{a}_{x_k,t}^{[t]}}\left(p_{x_k+t-1}^{[t-1]}+q_{x_k+t-1}^{[t-1]}\,G(t)\right)\times\exp\left(r_t^{\mathrm{PF}}-h^{[t-1]}\right)\\&=\mathrm{MEA}_k(t)\times\mathrm{IEA}_k(t).\end{aligned} \tag{6}$$

This version of $\mathrm{MEA}_k(t)$ contains an adjustment for changes in the valuation basis and a term related to the proportion of the group gain that is allocated to the k^{th} member (in this case, based on their death probability). Unlike the mortality adjustment found in Section 2.2, this factor depends on the member's death probability, making it cohort-specific—not shared by the whole pool anymore; that is, $\mathrm{MEA}_k(t)$ is different for two given members unless they have the same sex and age.

The investment experience adjustment factor is the same as that found in Section 2.2: it depends on the actual time-t return on the portfolio and the hurdle rate used at time $t-1$, $h^{[t-1]}$.

2.4 EXAMPLE AND COMPARISON BETWEEN THE TWO ADJUSTMENT FACTORS

We now consider a simple numerical example with two cohorts to illustrate the operation of these stylized lifetime pension pools and the two update rules above.[14] All calculations are based on unrounded numbers, but rounded numbers are presented in the text for ease of presentation.

We consider 100 new female members at time 0; half of the members are 65 years old, and the other half are 75 years old. Let us assume that mortality is modelled by the female Canadian Pensioners' Mortality (CPM) 2014 table with generational adjustments (Scale B) and that the flat (continuously compounded) hurdle rate is set to 4.5% for all t.[15] We also assume that the pool begins its activity in 2023; that is, 2023 is assumed to be time 0. The actuarial present values of the annuities due associated with these assumptions are $\ddot{a}_{65,0}^{[0]} = 15.0848$ and $\ddot{a}_{75,0}^{[0]} = 11.5469$.

If each member deposits \$1,000,000 in the pool, the total assets at inception are \$100,000,000. This initial asset value allows every 65-year-old member to receive a benefit of

$$B_k(0) = \frac{1{,}000{,}000}{15.0848} = 66{,}292, \quad \forall\, k \in \mathcal{L}_0 \text{ such that } x_k = 65,$$

and every 75-year-old member to receive a benefit of

$$B_k(0) = \frac{1{,}000{,}000}{11.5469} = 86{,}603, \quad \forall\, k \in \mathcal{L}_0 \text{ such that } x_k = 75$$

at inception. After paying these benefits, the total asset value drops to

$$100{,}000{,}000 - \sum_{k \in \mathcal{L}_0} B_k(0) = 100{,}000{,}000 - (50)(66{,}292) - (50)(86{,}603) = 92{,}355{,}236.$$

This amount is then invested for a year in a portfolio earning an uncertain rate of return. Suppose that the continuously compounded rate of return realized in the first year is $r_1^{\mathrm{PF}} = 3\%$; this leads to a time-1 asset value of

$$A(1) = 92{,}355{,}236 \times \exp(0.03) = 95{,}167{,}871.$$

In our adjustment factor calculation, we assume that five members die during the first year: two 65-year-old members and three 75-year-old members.[16]

2.4.1 GROUP-BASED ADJUSTMENTS

For the group-based adjustment, the time-1 post-redistribution benefit $B_k(1)$ for each surviving member is given by

$$B_k(0) \frac{A(1)}{\sum_{j \in \mathcal{L}_1} B_j(0)\, \ddot{a}_{x_j,1}^{[1]}} = B_k(0) \frac{95{,}167{,}871}{(50-2)(66{,}292)(14.8033) + (50-3)(86{,}603)(11.1708)}$$
$$= \begin{cases} 68{,}150 & \text{if } x_k = 65 \\ 89{,}031 & \text{if } x_k = 75 \end{cases},$$

[14] In practice, we would expect more members and potentially more cohorts, too; the full demographic model used in our analysis is presented in Section 4.
[15] See Appendix B for more details on the mortality modelling and the CPM 2014 table.
[16] Note that this is an extreme event for the purpose of illustration. The probability of this many or more deaths in a single year is less than 0.1%.

where $\ddot{a}^{[1]}_{65,1} = 14.8033$ and $\ddot{a}^{[1]}_{75,1} = 11.1708$ based on the CPM 2014 generational life table. In other words, the time-1 benefits are the time-1 asset values divided across the survivors (while accounting for their updated annuity factors).

We obtain identical adjustments when using the formula developed in Section 2.2:

$$\begin{aligned} \mathrm{MEA}(1) &= \left(\frac{\sum_{j \in \mathcal{L}_{0^+}} B_j(0)\, p^{[0]}_{x_j} \ddot{a}^{[0]}_{x_j,1}}{\sum_{j \in \mathcal{L}_1} B_j(0)\, \ddot{a}^{[1]}_{x_j,1}} \right) &= \left(\frac{96{,}606{,}149}{92{,}573{,}408} \right) &= 1.0436, \\ \mathrm{IEA}(1) &= \exp\left(r^{\mathrm{PF}}_1 - h^{[0]} \right) &= \exp(0.03 - 0.045) &= 0.9851, \end{aligned}$$

leading to a total adjustment factor of $\alpha(1) = \mathrm{MEA}(1) \times \mathrm{IEA}(1) =$ 1.0280, and a benefit of

$$B_k(1) = B_k(0)\, \alpha(1) = \begin{cases} 68{,}150 & \text{if } x_k = 65 \\ 89{,}031 & \text{if } x_k = 75 \end{cases}.$$

Even though investment returns were below the hurdle rate—which should have caused a decrease in the time-1 benefits—more members than expected died during the first year. This ultimately leads to an adjustment factor that is higher than one, meaning that the time-1 benefits are larger than those paid at time 0.

By applying the same logic recursively, one can obtain the benefit for each year and member once the realized mortality and rate of return are observed.

2.4.2 COHORT-BASED ADJUSTMENTS

To apply the cohort-based adjustments, we first need to compute the nominal yield for each cohort, which is as follows:

$$y_k(1) = \frac{q^{[0]}_{x_k}}{1 - q^{[0]}_{x_k}} = \begin{cases} 0.0048 & \text{if } x_k = 65 \\ 0.0126 & \text{if } x_k = 75 \end{cases},$$

where $q^{[0]}_{65} = 0.0047$ and $q^{[0]}_{75} = 0.0124$.

Then, based on the actual experience, we compute the group gain by dividing the sum of all balances forfeited by those who died during the year by the sum of the nominal gains of those who survived:

$$G(1) = \frac{\left(2(1{,}000{,}000 - 66{,}292) + 3(1{,}000{,}000 - 86{,}603)\right)}{\left(48(0.0048)(1{,}000{,}000 - 66{,}292) + 47(0.0126)(1{,}000{,}000 - 86{,}603)\right)} = 6.1246.$$

In this case, the group gain is quite sizeable: the assets forfeited by those who actually died are significantly more than the assets that were expected to be left behind. From the nominal yields and the group gain, we can obtain each member's account value by simply adding the k^{th} member's share of the decedents' forfeiture, as follows:

$$A_k(1) = (1{,}000{,}000 - 66{,}292)\exp(0.03)\left(1 + (0.0048)(6.1246)\right) = 990{,}222$$

if $x_k = 65$, and

$$A_k(1) = (1{,}000{,}000 - 86{,}603)\exp(0.03)\left(1 + (0.0126)(6.1246)\right) = 1{,}013{,}560$$

if $x_k = 75$, by simply adding the k^{th} member's share of the decedents' forfeiture.

Finally, by equating these asset values to the prospective value of the accounts of Equation (5), we get the following time-1 benefits:

$$B_k(t) = \frac{A_k(1)}{\ddot{a}^{[1]}_{x_k,1}} = \begin{cases} 66{,}892 & \text{if } x_k = 65 \\ 90{,}733 & \text{if } x_k = 75 \end{cases},$$

because $\ddot{a}^{[1]}_{65,1} = 14.8033$ and $\ddot{a}^{[1]}_{75,1} = 11.1708$.

Similar to Section 2.4.1, we can split the time-1 adjustment factors into a mortality experience adjustment and an investment experience adjustment:

$$\text{MEA}_k(1) = p^{[0]}_{x_k} + q^{[0]}_{x_k}\, G(1) = \begin{cases} 1.0243 & \text{if } x_k = 65 \\ 1.0635 & \text{if } x_k = 75 \end{cases}$$

and

$$\text{IEA}_k(1) = \exp\left(r^{\text{PF}}_1 - h^{[0]}\right) = 0.9851.$$

The investment experience adjustment is the same as that in Section 2.4.1. The mortality experience adjustment factors, however, are different: as the 75-year-old members are *riskier* and have larger nominal yields, they also get a larger piece of the group gain, increasing their mortality experience adjustments relative to the group-based adjustment of Section 2.4.1. For 65-year-old members, it is the opposite.

Note that, overall, the money that is distributed to survivors is the same in both cases (i.e., group-based and cohort-based adjustments). The former method applies the same adjustments to all members, thus not accounting for each member's risk profile. However, having the same factor for all members is easier to handle from a bookkeeping perspective and may be easier for members to understand. There is, therefore, a trade-off between these update rules.

Also, note that the entire mortality experience gain is being shared across the entire group of 100 members in both cases. This is different from breaking the group into smaller pools, with the 65-year-olds sharing in the gain from two deaths and the 75-year-olds sharing in the gain from three deaths.

2.5 A NOTE ON THE DISTRIBUTION OF MORTALITY EXPERIENCE ADJUSTMENT FACTORS

The expected value of the mortality experience adjustment factor applicable to a single cohort tends to one asymptotically as the size of the cohort approaches infinity and the probability of survival approaches one. However, this expected value may far exceed one when the cohort is small and the probability of death is non-negligible, leading to systematic increases in benefits at older ages.[17]

As an illustration, consider a lifetime pension pool with a single cohort consisting of ten members aged x. Suppose the probability of surviving one year is $p_x = 0.5$. Let N denote the number of survivors in the pool one year later. Then, N has a binomial distribution with $n = 10$ and probability of success p_x (see Appendix B for more details). The expected number of members in the pool one year later is five, but the actual number could be between zero and 10. Different numbers of survivors will give rise to different MEA factors, but the distribution of these MEA factors is no longer binomial. In fact, although the distribution of N is symmetric around five, the distribution of the mortality

[17] Note that this observation is true from the pool's perspective. The distribution of the MEA applicable to a specific member, and its expected value could be quite different when conditioning on that member's survival.

experience adjustment is highly skewed. If there are exactly five survivors, experience matches our assumption, so the mortality experience adjustment is exactly one. By contrast, if there are ten survivors, there are no forfeited assets from decedents, so the fund (which was expected to be divided among five survivors aged $x + 1$) now must be divided among ten members. As a result, every survivor's benefit will have to be cut in half, resulting in an MEA of 50%. At the other extreme, if there is only a single survivor, this member will inherit the entire fund instead of having to split it with four others, so her benefit will increase fivefold, giving rise to an MEA of 500%.[18] Other positive and negative outcomes will be similarly distorted, giving rise to a rather skewed distribution with an expected value of 1.14—a 14% expected increase in the benefit on account of mortality experience alone. This is entirely due to the asymmetry between the value of the MEA in the extreme right and extreme left tails of the distribution—that is, the impact of having fewer deaths than expected versus more than expected. Although the former leads to significant benefit reductions (50% in this case), the latter may bring considerably larger benefit increases (500%), pulling the expected value of mortality experience adjustments far above 100%.[19]

At younger ages, the probability of survival is higher, and the likelihood of extreme outcomes leading to very large MEA factors is much smaller. When p_x is 0.9, the largest possible MEA (corresponding to one survivor out of ten initial members) is still 500%, but the probability of this event occurring is significantly lower (9×10^{-9} compared to 0.01), so the expected MEA becomes 1.013. When p_x is 0.99, the expected MEA is 1.001. Thus, the distortion in model results does not extend to younger ages.

As the size of the initial group increases, even larger MEA factors become possible, but at the same time, all extreme events become much less likely. For example, with 20 initial members and a 50% survival probability, the largest possible MEA (corresponding to only one survivor) is 1,000% (a tenfold increase in the benefit) but the expected MEA is only 1.06. With 100 members, the expected MEA shrinks to 1.01. Therefore, the distortion in model results should not significantly impact open groups.

When there are several cohorts in the pool, the group-based MEA can be thought of as a weighted average of the individual cohorts' MEAs. Consequently, the distribution of the group MEA will be less skewed than that of the older cohorts on their own.

The skewness of the distribution of the MEA factor for a single elderly cohort has three practical implications. First, it may not always be the case that choosing best estimate assumptions for future mortality and investment experience leads to a level average benefit, particularly at advanced ages in a small pool. This should be considered in the design and operation of lifetime pension pools. Second, the skewness means that expected values for the cohort as a whole could differ significantly from the median. Summary statistics of projected lifetime pension pool outcomes for small groups at older ages should therefore be communicated carefully. Third, mixing multiple cohorts in the same lifetime pension pool can reduce the skewness of the mortality experience adjustments. If the goal is to keep the average adjustment relatively level, expanding the pool beyond a single cohort may help achieve this.

[18] We ignore the case where all ten members die, as this would extinguish the pool and make the calculation of an adjustment factor irrelevant.

[19] This is not to say that a given member of this pool can expect a 14% increase. If she survives, then her survival will tilt the balance towards a lower MEA. The expected number of other survivors (out of nine) is 4.5, so for any one member, the expected number of survivors, including themselves, is 5.5. This gives rise to a conditional expected benefit adjustment of one from the perspective of a member who survives. By contrast, the 14% expected increase is from the perspective of the pool operator, which is the focal point of this report, and reflects an unconditional expectation.

Section 3: Risk and Reward Measures

One of the main goals of this report is to compare lifetime pension pools' design elements, such as the hurdle rate policy, the potential for delayed recognition of gains and losses, and the possibility of adding new members after the pool's inception. To perform such comparisons, we employ different risk and reward measures. Similar measures were used by Sanders (2016) in the context of target benefit plans.

Throughout our report, we break down some of our results by age bands, each representing a retirement phase. Blanchett (2014) observes a "retirement spending smile" in aggregate data in the US, with spending in real terms being higher near retirement, then decreasing, and finally increasing again towards the end of life. In financial planning circles, this pattern is interpreted in the context of three distinct phases of retirement:[20]

1. The *go-go* years, when retirees have the freedom and ability to pursue many activities—this time is characterized by additional spending on travel, hobbies, and leisure.
2. The *slow-go* years, characterized by a slowdown in activities and travel, translate into more modest discretionary spending.
3. The *no-go* years, when health-related expenses tend to rise.

To account for these three phases, we split retirement into three different intervals: the go-go years, from retirement to 74 years old; the slow-go years, from 75 to 89 years old; and the no-go years, starting from 90 years old.[21] These three intervals are similar to those proposed by Davis (2019).

3.1 REWARD MEASURE

Reward measures, generally speaking, capture the potential *profit* an investment can produce. In the case of lifetime pension pools, members are paid benefits every year, and the average level of these benefits informs us about plausible rewards.

3.1.1 EXPECTED AVERAGE BENEFITS

Understanding benefit levels received by pool members throughout their lives is relevant to assessing the impact of pool design on *typical* benefits. In other words, this measure captures the general benefit trend over time and allows us to comment on whether benefits are adequate under *normal* circumstances. Potential deviations in the benefit stream from these averages are assessed in the following subsection.

Mathematically, the average benefit received by member k between times l and u is given by the expression

$$\overline{B}_k(l,u) = \frac{1}{u-l+1}\sum_{t=l}^{u} B_k(t)$$

for a given realization or scenario. Because this quantity is random and path-dependent, the expected average benefit is the mathematical expectation of $\overline{B}_k(l,u)$, defined as $\mathbb{E}\left[\overline{B}_k(l,u)\right]$. We compute this expected average benefit

[20] These phases are aligned with how the US health-care system is operated. Canadian retirees also go through similar phases, with health-related expenses potentially rising at advanced ages due to long-term care needs that are not covered under Canada's universal, publicly funded health-care system.

[21] We do not consider benefits for members older than 105 years old when splitting outcomes between the go-go, slow-go, and no-go years, as these benefits are extremely unstable—a by-product of the small pool size in the case of closed pools. Note that some other measures do consider horizons for benefits after age 105 (e.g., utility-based measures).

measure for each of the three phases of retirement mentioned above (i.e., go-go years, slow-go years, and no-go years).

3.2 RISK MEASURES

It is possible to select pool design elements that increase the expected average benefit. However, this would generally increase risk—the higher the potential reward, the higher the risk. For this reason, we introduce four measures that capture how risky the benefits are.

3.2.1 EXPECTED STANDARD DEVIATION OF ADJUSTMENTS

By construction, the adjustment factors capture the year-to-year changes in the benefits. Large deviations in these adjustments imply that members should expect very erratic benefit streams, whereas small deviations are synonymous with stable benefits.

For each scenario and a given member k, we compute the standard deviation of the benefit adjustments occurring between times l and u as follows:

$$\sigma_k(l,u) = \sqrt{\frac{1}{u-l+1}\sum_{t=l}^{u}\left(\alpha_k(t) - \overline{\alpha}_k(l,u)\right)^2},$$

where $\overline{\alpha}_k(l,u) = \frac{1}{u-l+1}\sum_{t=l}^{u}\alpha_k(t)$ is the average benefit adjustment over that period. The standard deviation reflects how variable the adjustment factors are when compared to their average level.

Note that this standard deviation is random: different possible realizations of mortality and financial variables give rise to different sequences of adjustment factors, leading to different standard deviations. We therefore take the mathematical expectation of $\sigma_k(l,u)$, $\mathbb{E}[\sigma_k(l,u)]$, as our measure. Again, we compute the expected standard deviation of adjustments for each of the three retirement phases.

3.2.2 EXPECTED STANDARD DEVIATION OF BENEFITS

The expected standard deviation of adjustments is concerned with the volatility of the year-on-year changes in the benefits over a given period. Yet some members, actuaries, and pool designers may care more about the volatility of the benefit levels themselves—not just variations in their adjustments over a given period. For this reason, we introduce a measure that focuses on benefits.

We compute the standard deviation of benefits received between times l and u as follows:

$$\zeta_k(l,u) = \sqrt{\frac{1}{u-l+1}\sum_{t=l}^{u}\left(B_k(t) - \overline{B}_k(l,u)\right)^2}$$

for each scenario and a given member k. Again, this standard deviation is random, and we compute its mathematical expectation (across possible scenarios) to obtain the measure—that is, $\mathbb{E}[\zeta_k(l,u)]$. It is computed for the three phases of retirement.

3.2.3 EXPECTED SHORTFALL PROBABILITY OF ADJUSTMENTS

An important characteristic of the standard deviation measures above is that they treat upside and downside volatility the same, whereas members may be more concerned about downside risk. To focus on unfavourable outcomes, we compute the occurrence of having adjustments below some threshold or comparator, denoted by C_k. This is interesting and somewhat different from measures that were previously introduced, as it allows us to understand how often we can expect benefit cuts—or benefit changes lower than some critical level—under a given design.

We compute the path- and member-specific shortfall probability of adjustments as follows:

$$p_k^{\alpha}(l,u) = \frac{1}{u-l+1}\sum_{t=l}^{u} \mathbf{1}_{\{\alpha_k(t)\, \leq\, C_k\}},$$

where $\mathbf{1}_{\{\alpha_k(t)\, \leq\, C_k\}}$ is an indicator that takes the value one if the time-t adjustment is less than or equal to the comparator, C_k, and zero otherwise. The measure then needs to be averaged across all scenarios, meaning that we need to calculate its mathematical expectation—that is, $\mathbb{E}[p_k^{\alpha}(l,u)]$.

In our report, we use two comparators C_k: 1.00 and 0.95. The first threshold focuses on benefit cuts relative to their current level. It is justified from a habit formation perspective: in the short term, members care most about shortfalls relative to the current level (see Pollak, 1970; MacDonald et al., 2013). Our second threshold is 0.95, referring to a drop of about 5% in the benefit, which would be alarming for most members. It is computed for the three phases of retirement: the go-go, slow-go, and no-go years.

3.2.4 EXPECTED SHORTFALL PROBABILITY OF BENEFITS

The standard deviation of the adjustment factors and their shortfall probabilities are interesting, as they inform us about the volatility of benefit payments from year to year and their risk of being too low. Yet the latter measure does not capture how often benefits are adequate when compared to some threshold or comparator—a targeted benefit C_k in this case.

For a given scenario and member k, we can define the shortfall probability of benefits as follows:

$$p_k^{B}(l,u) = \frac{1}{u-l+1}\sum_{t=l}^{u} \mathbf{1}_{\{B_k(t)\, \leq\, C_k\}}, \quad (7)$$

where $\mathbf{1}_{\{B_k(t)\, \leq\, C_k\}}$ is an indicator that takes the value one if the time-t benefit is less than or equal to the comparator C_k, and zero otherwise. Simply put, Equation (7) represents the proportion of time the benefits are less than some level C_k for a given realized scenario of mortality and investment returns.

Again, this quantity is random because it depends on the actual realization, so we compute the expected shortfall probability of benefits—that is, $\mathbb{E}[p_k(l,u)]$. Again, this can be done for go-go, slow-go, and no-go years. We focus on two different comparators: 100% of the initial benefit and 75% of the initial benefit.

3.3 HYBRID MEASURES

Reward or risk measures considered in isolation only convey part of the information about benefit streams: reward measures give information about the benefits members can expect under *normal* circumstances, whereas risk measures allow us to understand how uncertain these benefits are. Hybrid measures—accounting for both reward and risk dimensions—give a more complete picture of the risk–reward trade-off in lifetime pension pools. They, nonetheless, rely on an implicit weighting between reward and risk, loading differently on these two aspects. Because of this subjectivity, we present a few different hybrid measures.

3.3.1 MINIMUM AND AVERAGE BENEFITS AT RISK

Bégin and Sanders (2023) recently introduced a collection of measures to assess potential risk and reward in the context of lifetime pension pools: the *benefit at risk* (*BaR*). Like the value at risk, BaR has a probability level and a horizon, denoted by p and u, respectively. However, instead of investigating profits and losses, the focus is on benefits.

The notion of benefits is broad; one could be interested in benefits received at a specific point in time, whereas others could focus on the average benefit or the minimum benefit over a given period. We call this input the *benefit statistic* in our setting, and we denote the corresponding random variable by $\beta_k(u)$.

Another key input to the BaR is the comparator*,* denoted here by C_k, which acts as a benchmark for the benefit level $\beta_k(u)$. The comparator could be the current level of benefits when the BaR is being calculated, the expected future level of benefits, or the total lifetime benefits, for instance. This comparator captures the reward aspect of the BaR.

In all generality, the BaR at probability level p for benefit statistic $\beta_k(u)$ and comparator C_k is defined as

$$\mathrm{BaR}_p[C_k - \beta(u)] = F^{-1}_{C_k - \beta_k(u)}(p), \tag{8}$$

where $F^{-1}_{C_k - \beta_k(u)}(p)$ is the quantile function of the (random) difference between the comparator and the benefit statistic evaluated at p.[22] Taking a right-tail quantile of the distribution of the difference between the comparator and the benefit statistic helps us capture the risk dimension of the measure.

This risk–reward trade-off between the probability level and the comparator C_k allows us to define this measure as a hybrid between those of Sections 3.1 and 3.2. In other words, the probability level deals with how far we go in the right tail, or risk, and the comparator is representative of the benefits one should be expecting, or the reward.

The first specific measure considered in Bégin and Sanders (2023) is the *minimum BaR* (or *mBaR*). It tries to capture short-term benefit risk and reward, which is helpful in budgeting. The mBaR measure uses the initial level of benefit as the comparator C_k and the minimum benefit over a horizon of u years as the benefit statistic $\beta_k(u)$—that is,

$$\underline{B}_k(l, u) = \min_{t \in \{l,\ldots,u\}} B_k(t).$$

Mathematically, we replace the benefit statistic $\beta_k(u)$ with the minimum $\underline{B}_k(u)$ in Equation (8) to obtain

$$\mathrm{mBaR}(u) \equiv \mathrm{BaR}_p\left[B_k(0) - \underline{B}_k(1, u)\right] = F^{-1}_{B_k(0) - \underline{B}_k(1,u)}(p),$$

where $\mathrm{mBaR}(u)$ stands for the u-year minimum benefit at risk, while assuming the current benefit as the comparator. Bégin and Sanders (2023) considered a horizon of five years and a probability level of 97.5%. In this report, we will also consider this horizon and level, in addition to a longer horizon of ten years, along with a probability level of 95%.[23]

A second specific measure—better suited for decision-making—is considered in Bégin and Sanders (2023); it is called the *average BaR* (or *aBaR* for short). The relevant horizon for decision-making is longer than the budgeting horizon for mBaR.

[22] This function is also known as the percentile function or the percent-point function; it is the inverse of the cumulative distribution function (see Section 2 of Bégin and Sanders, 2023, for more details).
[23] There is a relationship between the probability level and the horizon considered. Indeed, a longer horizon should be coupled with a lower probability and a shorter horizon with a higher probability. As explained in Dhaene et al. (2008), a good approximation rule for the probability level suitable for a horizon u is $p = (p_{\text{annual}})^u$, where p_{annual} is the annual probability level. Note that assuming that $p_{\text{annual}} = 99.5\%$ (i.e., the level used for solvency capital requirements under Solvency II) yields an approximate five-year level of about 97.5% and an approximate ten-year level of about 95%—the levels selected for the mBaR.

Over a longer horizon, there are two main reasons the benefits may change:

1. The arrangement may target a non-level (i.e., either increasing or decreasing) expected benefit pattern.
2. There is statistical uncertainty around this target (i.e., actual outcomes will differ from expected levels, regardless of whether the expected benefits have an increasing, level, or decreasing pattern).

To consider both of these aspects simultaneously, we focus on the average benefit level over the entire horizon and compare the *actual average* benefit against the *expected average* benefit. In other words, the average observed benefit over a given horizon of u years, $\overline{B}_k(1,u)$, becomes the benefit statistic, and the expected average benefit, $\mathbb{E}\left[\overline{B}_k(1,u)\right]$, becomes the comparator.

Replacing the benefit statistic $\beta_k(u)$ by the average $\overline{B}_k(1,u)$ and the comparator C_k with $\mathbb{E}\left[\overline{B}_k(1,u)\right]$ in Equation (8) yields

$$\mathrm{aBaR(u)} \equiv \mathrm{BaR}_p\left[\mathbb{E}[\bar{B}(1,u)] - \overline{B}_k(1,u)\right] = F^{-1}_{\mathbb{E}[\bar{B}(1,u)]-\bar{B}(1,u)}(p),$$

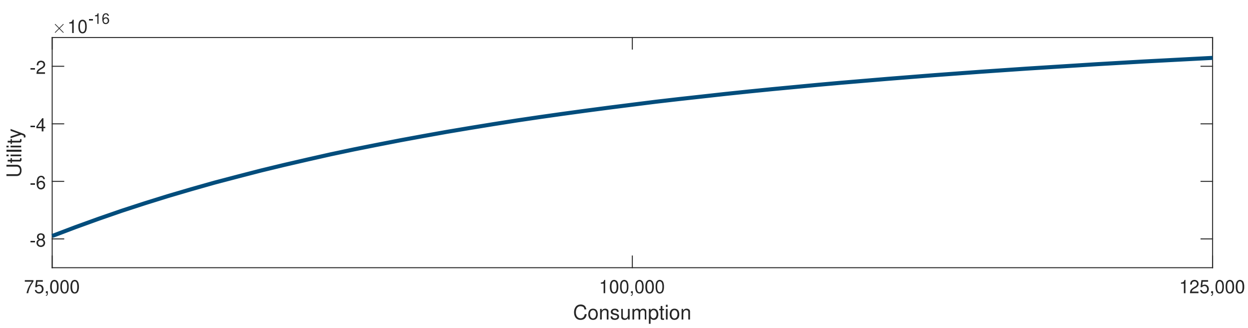
where $\mathrm{aBaR(u)}$ stands for the u-year average benefit at risk, while assuming a probability level of p. Since the aBaR measure focuses more on the long term, we select 20-, 30-, and 40-year horizons in the following sections. The probability levels are selected as 90%, 85%, and 80%, respectively, roughly in accordance with Footnote 23.

3.3.2 UTILITY-BASED CERTAINTY EQUIVALENT CONSUMPTION

Expected utility theory allows us to rank uncertain prospects based on individuals' preferences expressed through a utility (or preference) function. It is a helpful way to represent how members make decisions under risk; see, for instance, Stevenson (2019) for more details on utility theory and preference functions in the context of actuarial science.[24]

Figure 1.

ISOELASTIC UTILITY AS A FUNCTION OF CONSUMPTION.



Notes: This figure reports utility as a function of consumption. We set η to 4 in this figure, consistent with the value found by Barsky et al. (1997).

In this report, the isoelastic utility function (or power utility) is used to express utility in terms of consumption—in this case, the benefits. We select this specific function because it is commonly used in finance and financial economics. It is given by the following function of consumption, c:

[24] See also Gerber and Pafumi (1998) for an application of utility functions in an actuarial context.

$$u(c) = \frac{c^{1-\eta}}{1-\eta}, \quad \text{such that } \eta > 0 \text{ and } \eta \neq 1,$$

where η is a parameter representing the degree of relative risk aversion. We expect this parameter η to be larger than zero for risk-averse members.[25] Figure 1 reports an example of the isoelastic utility function.

Because each member receives a stream of benefits—rather than just a lump sum—we must aggregate consumption over the relevant period. In the present report, we use the following expected discounted utility model to do so:

$$\mathrm{EU}_k(u) = \sum_{t=1}^{u} \delta^{-t}\, u\big(B_k(t)\big),$$

where δ is a one-period discount factor associated with the member's subjective time preference.[26]

The expected discounted utility is, by itself, not informative unless it is transformed into a meaningful quantity. For this reason, many authors rely on certainty equivalent consumption (CEC) to compare random consumption streams. In our case, the k^{th} member's CEC—assuming this member survives until time u—is defined as the solution to the following equation:

$$\sum_{t=1}^{u} \delta^{-t}\, u\big(\mathrm{CEC}_k(u)\big) = \mathrm{EU}_k(u) \quad \Rightarrow \quad \mathrm{CEC}_k(u) = \left(\frac{\mathrm{EU}_k(u) \times (1-\eta)}{\sum_{t=1}^{u} \delta^{-t}}\right)^{\frac{1}{1-\eta}}.$$

Simply put, it is the level consumption stream that, if received with certainty, would produce the same (expected) discounted utility over the period in question as the random benefit stream does. We set η to 4, which is close to the value found by Barsky et al. (1997) in an empirical survey study relating to health and retirement. The subjective discount factor δ is set to $\exp(-\bar{r})$, where $\bar{r}$ is the long-run average level of the short rate, as given by the economic scenario generator (ESG) described in Appendix A. We consider four different time horizons u in this report—that is, 20, 30, 40, and 50 years. The last horizon corresponds to a CEC value for someone surviving until the end of the life table; this is obviously an extreme scenario, but it is relevant, as benefits may be quite uncertain for very old members.

The CEC measure, like the BaR, captures a trade-off between risk and reward. Indeed, Figure 1 shows that utility is increasing as a function of consumption—in this case, the benefit—but the added utility of each extra unit of consumption is smaller and smaller, leading to risk-averse behaviour.

[25] A parameter η of zero would lead to a risk-neutral member, whereas a parameter that is less than zero would be used to model the behaviour of a risk-seeking individual.
[26] A decision-maker's time preference indicates how much more they value receiving payments sooner than later; in other words, it indicates how impatient they are.

Section 4: Closed and Open Pools

This section investigates the difference between closed and open pools. The former arrangement takes in members only at inception; that is, no new members are permitted to join after inception, so the total number of members decreases as time passes. The open pool, on the other hand, allows for new members to enter the pool every year.

We first define the setup and the assumptions used to compute our various measures introduced in Section 3. Then we apply them to our base case and comment on the main differences between the two designs. We finally run four types of robustness tests in Sections 4.3 to 4.6: we investigate the impact of the pool size and age distribution as well as the consequences of changing the asset allocation policy and the sets of initial conditions on closed and open pools. Our tests consider both the group- and cohort-based adjustments introduced in Section 2.

4.1 SETUP AND ASSUMPTIONS

We assume that 1,000 female members join the plan at inception, in January 2023. Their mortality is modelled by the female CPM 2014 table with generational adjustments as per Scale B; see Appendix B for more details. The age distribution of these members depends on whether the pool is closed or open:

1. Closed pool: All members are 65 years old at inception in the closed pool.[27]
2. Open pool: The open pool's membership at inception is assumed to be stationary (i.e., as if the plan has been operating for a very long time with a consistent new entrant profile). Then, each year after inception, new members, all aged 65, join the plan so that the total number of pool members stays close to 1,000.

Each 65-year-old member brings in a total of \$1,000,000 at inception. For members joining the pool after inception in the open pool design, the initial asset value is increased for inflation to produce comparable benefits at age 65 in real terms. For members older than 65 at inception in the open pool, two adjustments are made. First, the asset value is reduced for inflation to maintain consistency in real terms. Second, an adjustment is made to reflect the fewer remaining payments based on the member's age at inception. Specifically, the initial asset value of the k^{th} member at inception is given as follows:

$$A_k(0) = \$1{,}000{,}000\,\exp\big(-\bar{q}(x_k - 65)\big)\,\frac{\ddot{a}^{[0]}_{x_k,0}}{\ddot{a}^{[0]}_{65,0}},$$

where $\bar{q}$ is the average annual inflation rate as given by the ESG of Appendix A (i.e., set to about 2% in this study).

For simplicity, we assume that the pool invests in only two assets: a stock index (stock index or S henceforth), acting as a proxy for equity market investments, and a Treasury bond (bond portfolio or B subsequently) with a duration of seven years, as a proxy for fixed-income investments.[28] The time-t stock index return, $R_t^{(\mathrm{S})}$, accounts for the index price change and the dividend payments, whereas the time-t bond portfolio return, $R_t^{(\mathrm{B})}$, relies on changes in the term structure of risk-free interest rates. Specifically, these two returns are computed as follows from the (monthly) outputs of our ESG:

[27] In one of the robustness tests of Section 4.3, we investigate a closed pool starting from the same stationary membership as the open pool.

[28] This choice is made for simplicity. Having an actual bond portfolio would be more realistic in this context (see, e.g., Fullmer and Sabin, 2019b, for an analysis of the impact of a properly constructed bond portfolio on lifetime pension pool payouts), but our simpler assumption should not have a qualitative impact on the bulk of our results (i.e., the numbers might change, but the general conclusions and results will stay the same).

$$R_t^{(S)} = \exp\left(\sum_{t'=12(t-1)+1}^{12t} \left(y_{t'} + \log\left(1 + \frac{d_{t'}}{12}\right)\right)\right) \qquad \text{and} \qquad R_t^{(B)} = \frac{\exp(-6\, r_{6,12t})}{\exp(-7\, r_{7,12(t-1)})},$$

where $y_{t'}$ is the stock index return for month t', $d_{t'}$ is the dividend yield for month t', and $r_{T,t'}$ is the T-year (risk-free) interest rate for month t' (see Appendix A for more details).[29] Assuming that the proportion invested in the stock index is set to ω, the time-t portfolio return is given by:

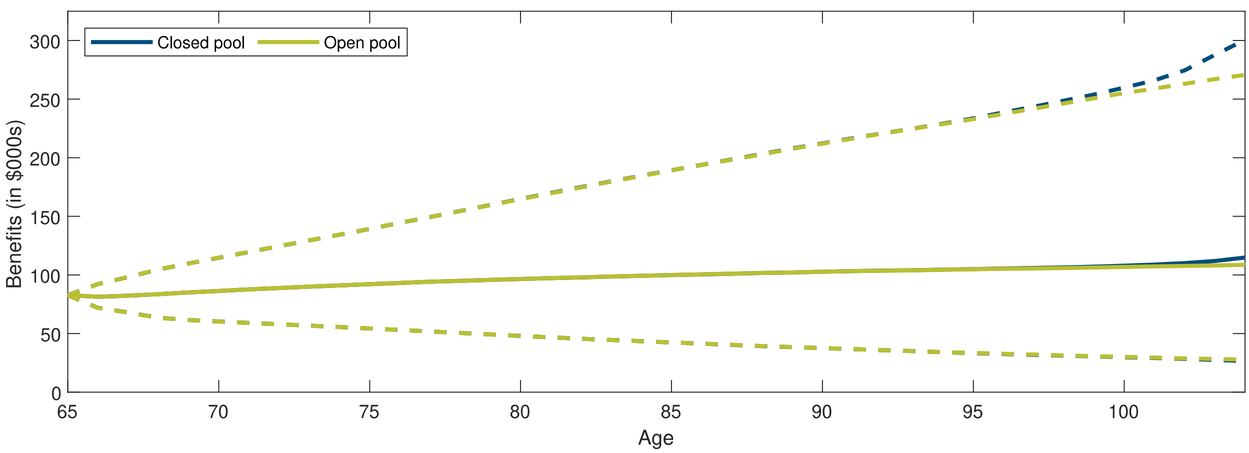
$$r_t^{\mathrm{PF}} = \log(\omega\, R_t^{(S)} + (1-\omega)\, R_t^{(B)}).$$

In our base case (and throughout the report unless otherwise stated), we select a 50–50 split, with half of the pool's assets invested in the stock index and the other half in the bond portfolio.

In this section, the hurdle rate is assumed constant; that is, $h^{[t]} = h$ for all t. We set it to the long-run (ultimate) median of the portfolio return such that there is no predictable escalation in the benefit payments in the long term.[30] This leads to an initial benefit of $82,758 for members aged 65 at inception.

Figure 2.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR CLOSED AND OPEN POOLS WITH GROUP ADJUSTMENTS.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both closed and open pools. We show the average (solid line) as well as 5th and 95th quantiles (dashed lines). We do so for two different types of pools. First, we consider a closed pool (in blue) with 1,000 65-year-old members; second, we assume an open pool (in green) where the pool's membership is assumed to be stationary (i.e., a membership consistent with a plan operating for a very long time). This figure relies on group adjustments presented in Section 2.2.

However, there might be short-term trends in benefits due to mismatches between the current average portfolio returns and the long-term average portfolio returns. For instance, in a low-interest rate environment, we expect interest rates to increase over time as they approach the stationary distribution on which the hurdle rate is based; this will give rise to a decreasing average benefit pattern in the short term. On the other hand, if initial interest rates are higher than their steady states (as in the base case), then we expect they will decrease over time (and the benefits will increase in the short term).

[29] In the present report, we ignore credit risk because the Canadian high-quality bond market is dominated by federal and provincial bonds.

[30] The hurdle rate is set to the long-term (ultimate) median portfolio return for years after short-term effects have worn off. In other words, it is the median from the stationary starting point.

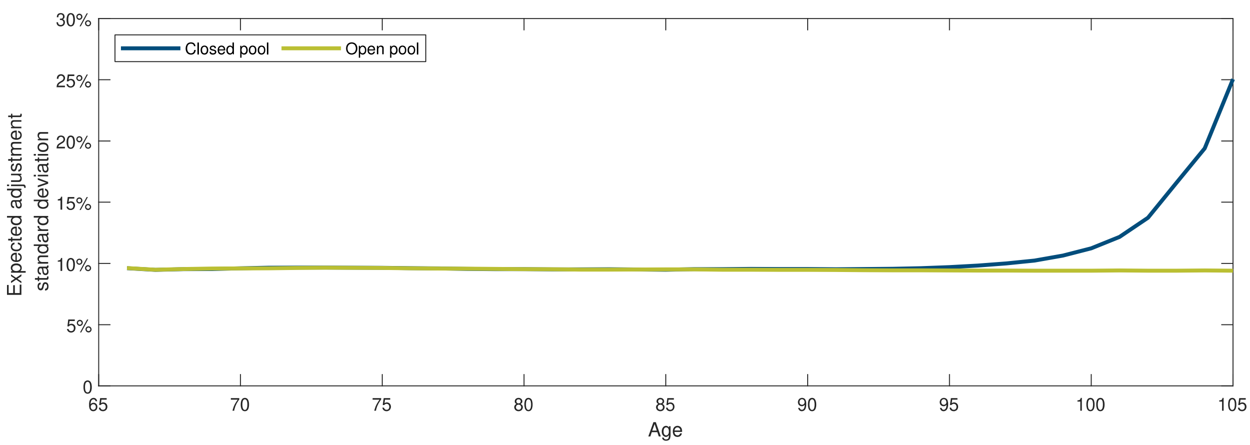
We rely on 25,000 scenarios generated from the ESG and the mortality model. For each scenario, we generate all financial and economic values in addition to decedents and the resulting pool membership in each year.

4.2 RESULTS ON CLOSED AND OPEN POOLS

Table 1 reports the various reward, risk, and hybrid measures for closed and open pools using the assumptions stated above. We report these measures for both the group- and cohort-based adjustments considered in Section 2 of this report.

Most measures are similar across all four cases under study—the open and closed pools with the group- and cohort-based adjustments. This is especially true for measures over the short term; that is, values calculated for the go-go and slow-go years as well as all values with horizons of less than 40 years. The difference between closed and open pools becomes more noticeable when considering older ages and measures over longer terms. Figure 2 reports the funnels of doubt (average value and 5^{th} and 95^{th} quantiles) of the annual benefits for both closed and open pools with group adjustments. At first, the average benefits increase due to an expected decrease in interest rate over the next ten years following 2023.[31] They are then somewhat constant for the first 10–20 years, when interest rates return to their long-run levels, on average. The funnels are very similar to one another when members are younger than 95 years old, explaining why the measures behave similarly for shorter horizons. We obtain a different picture when looking at longer terms.

Figure 3.
EXPECTED STANDARD DEVIATION OF ADJUSTMENTS AS A FUNCTION OF AGE FOR CLOSED AND OPEN POOLS.



Notes: This figure reports the expected adjustment standard deviation for ages 66 to 105 and for the group-based adjustments of a 65-year-old member at inception. This expected standard deviation is computed for both closed (in blue) and open (in green) pools. We assume 1,000 members in the former case, with no members entering the plan after inception (i.e., in 2023). The latter case also starts with 1,000 members; this pool's membership is assumed to be stationary (i.e., a membership consistent with a plan operating for a very long time). Members, aged 65 years old, also join the open pool every year so that the pool size stays somewhat constant over time.

The expected average benefit for the no-go years is slightly higher for closed pools when compared to open pools from the pool's perspective.[32] This is because the expected mortality experience adjustment is higher than one in a

[31] Note that interest rates were high in Canada at the beginning of 2023, following the COVID-19 pandemic, when compared to the rates observed over the last thirty years, and that these rates are expected to decrease over time. In our ESG, their average decreases over the next 10–20 years, as expected.
[32] Note that this result could be quite different if one were to look at average benefits from a surviving member's perspective.

closed pool when the number of members is low, and the expected mortality rate is substantial, as illustrated in Section 2.5.

Table 1.
REWARD, RISK, AND HYBRID MEASURES FOR CLOSED AND OPEN POOLS.

Measures	Group		Cohort	
	Closed pool	Open pool	Closed pool	Open pool
Expected average benefits (in $000s)				
Go-go years	86.9	86.9	86.9	86.9
Slow-go years	98.4	98.4	98.4	98.4
No-go years	108.0	106.1	108.0	106.4
Expected standard deviation of adjustments				
Go-go years	9.3%	9.3%	9.3%	9.3%
Slow-go years	9.2%	9.1%	9.2%	9.2%
No-go years	12.1%	9.0%	12.1%	10.4%
Expected standard deviation of benefits (in $000s)				
Go-go years	10.9	10.9	10.9	10.9
Slow-go years	15.6	15.6	15.6	15.6
No-go years	21.3	16.7	21.3	18.5
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	45%	45%	45%	45%
Slow-go years	48%	48%	48%	48%
No-go years	50%	50%	50%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	25%	25%	25%	25%
Slow-go years	27%	27%	27%	27%
No-go years	31%	28%	31%	31%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	44%	44%	44%	44%
Slow-go years	41%	41%	41%	41%
No-go years	46%	47%	46%	47%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$				
Go-go years	7%	7%	7%	7%
Slow-go years	18%	18%	18%	18%
No-go years	29%	29%	29%	29%
Minimum benefit at risk (in $000s)				
5-year horizon	29.4	29.4	29.4	29.4
10-year horizon	32.2	32.2	32.2	32.2
Average benefit at risk (in $000s)				
20-year horizon	26.9	26.8	26.9	26.8
30-year horizon	30.0	30.1	30.0	30.0
40-year horizon	32.7	32.2	32.7	32.4
Certainty equivalent consumption (in $000s)				
20-year horizon	74.8	74.8	74.8	74.8
30-year horizon	68.0	68.1	68.0	68.0
40-year horizon	60.4	61.1	60.4	59.4
50-year horizon	8.3	53.7	8.3	46.0

Notes: This table reports the various measures introduced in Section 3 for both closed and open pools and for group- and cohort-based adjustments. We consider pools of 1,000 members at inception in this table. “Group” stands for group-based adjustments, and “cohort” for cohort-specific adjustments.

This increase in the expected average benefit in the no-go years comes at a high cost, however: the expected standard deviations of adjustments and benefits are significantly higher for closed pools than for open pools. For closed pools, the expected standard deviation of adjustments goes from about 9.3% during go-go and 9.2% during slow-go years to 12.1% during the no-go years—a relative increase of about 31%. It stays rather constant for open pools with group-based adjustments, whereas it increases from 9.2% during slow-go years to 10.4% during no-go years for cohort-specific adjustments. The increase for the cohort-based adjustments is expected, to some extent, as nominal yields for older ages are higher—a by-product of the increase in the death probabilities. Figure 3 reports the expected standard deviations of adjustment as a function of age for both closed and open pools in the context of group-based adjustments. Indeed, these expected standard deviations of adjustment are similar for closed and open pools for ages below 90 but increase for older ages in the case of closed pools. Thus, a risk–reward trade-off exists between (tiny) additional rewards (i.e., increases in the expected average benefits) and additional risk (i.e., increases in the expected standard deviation of adjustments). Hybrid measures help us comment on this trade-off.

The aBaR for a horizon of 40 years is slightly lower (that is, better) for open pools, especially in the case of group-based adjustments. All in all, we observe a decrease of $500 ($300) for open pools with group (cohort) adjustments—a drop (that is, an improvement) of 1.5% (0.9%) in the 40-year average BaR.

CECs display similar results: even though the values are similar for 20- and 30-year horizons across the four designs, they tend to differ for 40- and 50-year horizons. Specifically for the group-based adjustments, the CEC increases by $700 when considering open versus closed pools over a 40-year horizon. The difference between closed and open pools becomes huge over a 50-year horizon: the uncertainty in closed pools reaches very high levels, making the certainty equivalent consumption values minuscule.

Different demographic profiles—implied by open and closed pools—can also impact the pool's asset allocation to some extent. Indeed, having an open pool allows for additional risk capacity that would not be possible without the presence of young members in the pool; this gives older members the opportunity to invest in riskier assets. It also introduces intergenerational risk sharing between generations in the pool, which may potentially bring additional challenges concerning fairness. So having an open pool is beneficial, especially for older members, but one should investigate the key issues of intergenerational risk sharing and fairness before considering open pools if these concerns are important for the pool and its members.

Note that this section is also related to a broader conversation on lifetime pension pool membership. For instance, Bernhardt and Donnelly (2021) investigated the trade-off between income stability and the number of members in the pool. More recently, Donnelly (2023) looked into heterogeneity and concluded that its effect tends to be generally small in the context of lifetime pension pools.

The rest of this section is devoted to additional tests and robustness analyses.

4.3 IMPACT OF POOL SIZE

The results of the previous section relied on pools of 1,000 members at inception. This section assesses the impact of decreasing the pool size to 100 and 500 members at inception.

Table 2 reports the results for these two pool sizes; we again rely on two different update rules (i.e., group- and cohort-based adjustments) and show results for closed and open pools, as defined above. Overall, the results for the go-go and slow-go years as well as those for horizons of 5, 10, 20, and 30 years are similar across the different pool sizes and virtually identical to those presented in Table 1. The difference between open and closed pools is becoming more evident for longer horizons (i.e., the no-go years as well as 40- and 50-year horizons).

The expected average benefits tend to be slightly higher for closed pools and increase as the number of pool members decreases, consistent with the derivations of Section 2.5. For a small pool, one additional decedent can significantly raise the benefit level, explaining the higher average benefit. This comes at a high cost, however: the uncertainty associated with adjustments and benefits is also higher for closed pools when compared to open ones. For instance, for group-based adjustments during the no-go years, the expected standard deviation of adjustments goes from 12.1% with pools of 1,000 members at inception to 15.4% with pools of 500 members, and to 26.0% with pools that begin with 100 members. For open pools, on the other hand, this standard deviation stays rather constant—around 9.0%. For the cohort-based adjustments, we see a similar story, with even higher standard deviations for small pools.

Table 2.

REWARD, RISK, AND HYBRID MEASURES FOR CLOSED AND OPEN POOLS WITH DIFFERENT POOL SIZES.

	Pool size of 100 at inception				Pool size of 500 at inception			
	Group		Cohort		Group		Cohort	
Measures	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool
Expected average benefits (in $000s)								
Go-go years	86.9	86.9	86.9	86.9	86.9	86.9	86.9	86.9
Slow-go years	98.7	98.7	98.7	98.8	98.5	98.4	98.5	98.5
No-go years	115.6	106.6	115.6	110.2	110.3	106.1	110.3	106.7
Expected standard deviation of adjustments								
Go-go years	9.4%	9.4%	9.4%	9.4%	9.3%	9.4%	9.3%	9.3%
Slow-go years	9.4%	9.2%	9.4%	9.4%	9.2%	9.1%	9.2%	9.2%
No-go years	26.0%	9.1%	26.0%	18.7%	15.4%	9.0%	15.4%	11.5%
Expected standard deviation of benefits (in $000s)								
Go-go years	10.9	11.0	10.9	10.9	10.9	10.9	10.9	10.9
Slow-go years	15.9	15.8	15.9	16.0	15.7	15.6	15.7	15.7
No-go years	39.7	16.9	39.7	32.0	27.0	16.7	27.0	20.3
Expected shortfall probability of adjustments, $C_k = 1.00$								
Go-go years	45%	45%	45%	45%	45%	45%	45%	45%
Slow-go years	48%	48%	48%	48%	48%	48%	48%	48%
No-go years	50%	50%	50%	54%	50%	50%	50%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$								
Go-go years	25%	25%	25%	25%	25%	25%	25%	25%
Slow-go years	27%	27%	27%	27%	27%	27%	27%	27%
No-go years	37%	28%	37%	42%	33%	28%	33%	33%
Expected shortfall probability of benefits, $C_k = B_k(0)$								
Go-go years	45%	44%	45%	44%	44%	44%	44%	44%
Slow-go years	41%	41%	41%	41%	41%	41%	41%	41%
No-go years	45%	46%	45%	48%	46%	47%	46%	47%
Expected shortfall probability of benefits, $C_k = 0.75\ B_k(0)$								
Go-go years	7%	7%	7%	7%	7%	7%	7%	7%
Slow-go years	18%	18%	18%	18%	18%	18%	18%	18%
No-go years	29%	29%	29%	32%	29%	29%	29%	30%
Minimum benefit at risk (in $000s)								
5-year horizon	29.6	29.8	29.6	29.5	29.5	29.5	29.5	29.4
10-year horizon	32.3	32.5	32.3	32.3	32.2	32.2	32.2	32.3
Average benefit at risk (in $000s)								
20-year horizon	26.9	27.0	26.9	26.9	26.8	26.9	26.8	26.8
30-year horizon	30.3	30.3	30.3	30.4	30.1	30.1	30.1	30.1
40-year horizon	34.3	32.5	34.3	34.3	33.1	32.2	33.1	32.4
Certainty equivalent consumption (in $000s)								
20-year horizon	74.7	74.7	74.7	74.8	74.8	74.8	74.8	74.8
30-year horizon	67.8	67.9	67.8	67.1	68.0	68.1	68.0	67.9
40-year horizon	55.1	60.8	55.1	39.2	59.8	61.1	59.8	58.1
50-year horizon	1.4	53.4	1.4	3.5	5.0	53.7	5.0	35.7

Notes: This table reports the various measures introduced in Section 3 for both closed and open pools and for group- and cohort-based adjustments. We consider pools of 100 and 500 members at inception in this table. “Group” stands for group-based adjustments, and “cohort” for cohort-specific adjustments.

The expected shortfall probabilities for pools of 100 and 500 members tend to be similar throughout the cases, except for longer horizons. The behaviour of group- and cohort-based adjustments is different for small pools:

Table 3.

REWARD, RISK, AND HYBRID MEASURES FOR CLOSED AND OPEN POOLS WITH STATIONARY MEMBERSHIP AT INCEPTION IN THE CLOSED POOL.

Measures	Group		Cohort	
	Closed pool	Open pool	Closed pool	Open pool
Expected average benefits (in $000s)				
Go-go years	86.9	86.9	86.9	86.9
Slow-go years	98.5	98.4	98.4	98.4
No-go years	114.4	106.1	113.3	106.4
Expected standard deviation of adjustments				
Go-go years	9.4%	9.3%	9.3%	9.3%
Slow-go years	9.3%	9.1%	9.2%	9.2%
No-go years	25.7%	9.0%	24.0%	10.4%
Expected standard deviation of benefits (in $000s)				
Go-go years	10.9	10.9	10.9	10.9
Slow-go years	15.8	15.6	15.6	15.6
No-go years	38.6	16.7	35.7	18.5
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	45%	45%	45%	45%
Slow-go years	48%	48%	48%	48%
No-go years	50%	50%	50%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	25%	25%	25%	25%
Slow-go years	27%	27%	27%	27%
No-go years	37%	28%	36%	31%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	44%	44%	44%	44%
Slow-go years	41%	41%	41%	41%
No-go years	45%	47%	44%	47%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$				
Go-go years	7%	7%	7%	7%
Slow-go years	18%	18%	18%	18%
No-go years	29%	29%	28%	29%
Minimum benefit at risk (in $000s)				
5-year horizon	29.5	29.4	29.4	29.4
10-year horizon	32.3	32.2	32.3	32.2
Average benefit at risk (in $000s)				
20-year horizon	26.9	26.8	26.8	26.8
30-year horizon	30.3	30.1	30.1	30.0
40-year horizon	34.0	32.2	33.6	32.4
Certainty equivalent consumption (in $000s)				
20-year horizon	74.8	74.8	74.8	74.8
30-year horizon	68.0	68.1	68.1	68.0
40-year horizon	54.0	61.1	56.1	59.4
50-year horizon	4.5	53.7	4.7	46.0

Notes: This table reports the various measures introduced in Section 3 for both closed and open pools and for group- and cohort-based adjustments. The closed pool considers an initial membership that is stationary (i.e., a membership consistent with a plan operating for a very long time). We consider pools of 1,000 members at inception in this table. “Group” stands for group-based adjustments, and “cohort” for cohort-specific adjustments.

1. For group-based adjustments: The closed pool exhibits higher expected shortfall probabilities of adjustments than the open pool. The expected probability is 37% for closed pools of 100 members when the threshold is set to 0.95; it is 28% for open pools of the same size.
2. For cohort-based adjustments: The open pool exhibits higher expected shortfall probabilities of adjustments when compared to the closed pool. The 100-member closed pool has an expected shortfall probability of

adjustments of 37% when the threshold is 0.95; it is 42% for the same open pool. This is mainly explained by the members' age heterogeneity in the open pool and its impact on the group gain.

Table 4.

REWARD, RISK, AND HYBRID MEASURES FOR CLOSED AND OPEN POOLS WITH DIFFERENT ASSET ALLOCATION STRATEGIES.

	25% invested in stock index				75% invested in stock index			
	Group		Cohort		Group		Cohort	
Measures	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool
Expected average benefits (in $000s)								
Go-go years	71.3	71.4	71.3	71.3	102.5	102.5	102.5	102.5
Slow-go years	84.9	84.9	84.9	84.9	113.1	113.1	113.1	113.1
No-go years	99.8	98.0	99.8	98.3	119.1	117.0	119.1	117.4
Expected standard deviation of adjustments								
Go-go years	8.4%	8.4%	8.4%	8.4%	11.8%	11.8%	11.8%	11.8%
Slow-go years	8.0%	8.0%	8.0%	8.0%	11.7%	11.7%	11.7%	11.7%
No-go years	11.1%	7.8%	11.1%	9.2%	14.2%	11.6%	14.2%	12.8%
Expected standard deviation of benefits (in $000s)								
Go-go years	7.8	7.8	7.8	7.8	16.2	16.2	16.2	16.2
Slow-go years	12.3	12.3	12.3	12.3	22.0	21.9	22.0	22.0
No-go years	18.7	13.9	18.7	15.8	27.1	22.7	27.1	24.4
Expected shortfall probability of adjustments, $C_k = 1.00$								
Go-go years	46%	46%	46%	46%	45%	45%	45%	45%
Slow-go years	49%	49%	49%	49%	48%	48%	48%	48%
No-go years	50%	51%	50%	51%	50%	50%	50%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$								
Go-go years	23%	23%	23%	23%	28%	28%	28%	28%
Slow-go years	23%	23%	23%	23%	31%	31%	31%	31%
No-go years	28%	24%	28%	28%	34%	32%	34%	34%
Expected shortfall probability of benefits, $C_k = B_k(0)$								
Go-go years	45%	45%	45%	45%	44%	44%	44%	44%
Slow-go years	32%	32%	32%	32%	48%	48%	48%	48%
No-go years	35%	35%	35%	35%	54%	55%	54%	55%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$								
Go-go years	1%	1%	1%	1%	13%	13%	13%	13%
Slow-go years	7%	7%	7%	7%	28%	28%	28%	28%
No-go years	16%	16%	16%	17%	40%	40%	40%	40%
Minimum benefit at risk (in $000s)								
5-year horizon	18.1	18.1	18.1	18.1	45.1	45.1	45.1	45.2
10-year horizon	18.5	18.5	18.5	18.5	49.8	49.7	49.8	49.9
Average benefit at risk (in $000s)								
20-year horizon	14.8	14.8	14.8	14.8	42.2	42.2	42.2	42.3
30-year horizon	19.2	19.2	19.2	19.2	45.2	45.2	45.2	45.2
40-year horizon	23.0	22.6	23.0	22.8	47.1	46.6	47.1	46.7
Certainty equivalent consumption (in $000s)								
20-year horizon	69.4	69.4	69.4	69.4	67.7	67.7	67.7	67.7
30-year horizon	68.3	68.3	68.3	68.3	52.2	52.3	52.2	51.9
40-year horizon	65.9	66.3	65.9	65.5	40.1	40.6	40.1	37.7
50-year horizon	12.9	63.9	12.9	59.5	2.6	29.2	2.6	20.7

Notes: This table reports the various measures introduced in Section 3 for both closed and open pools and for group- and cohort-based adjustments. We consider pools of 1,000 members at inception in this table. "Group" stands for group-based adjustments, and "cohort" for cohort-specific adjustments.

The mBaR measures are very similar across all cases; the aBaR measures, on the other hand, are different. Consistent with the 1,000-member pool, the aBaR tends to be larger for closed pools when compared to open ones for a horizon of 40 years, and this difference is more significant when dealing with small pools.

Finally, the CEC measures exhibit a similar story to that of the aBaRs for long horizons: a small open pool performs better from a utility standpoint than a small closed pool. The group-based adjustment yields better results than the cohort one for 40-year horizons, again due to the mortality experience adjustment uncertainty, which increases over time for the cohort method. Note that as the pool size increases, the cohort-based adjustment tends to yield results similar to those using group-based adjustments.

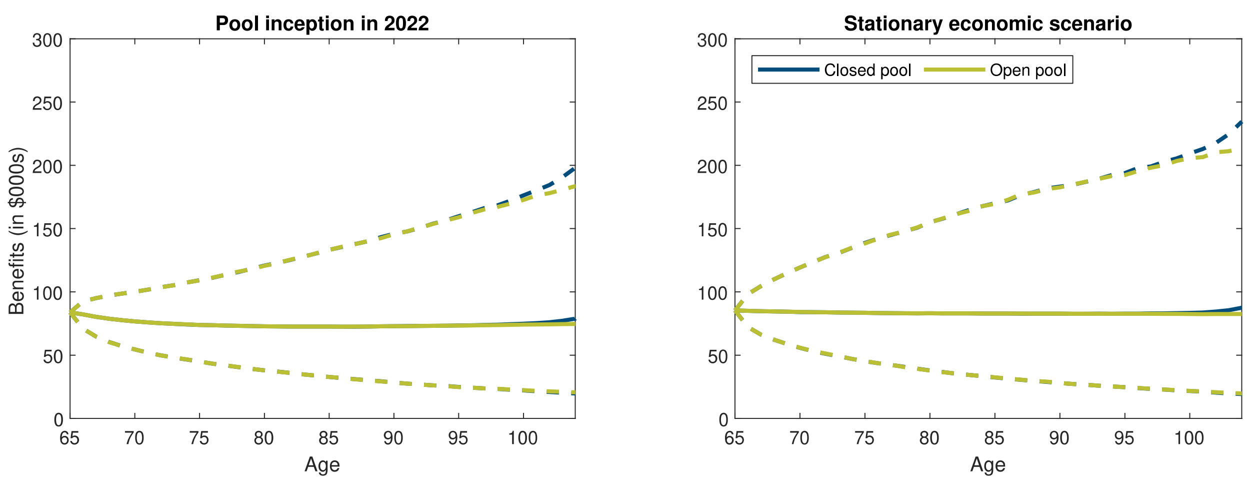
4.4 IMPACT OF AGE DISTRIBUTION

Tables 1 and 2 assumed that all members have the same age at inception in the closed pool (i.e., 65 years old). Another possible assumption for the closed pool would be having an age distribution at inception that is consistent with the initial (stationary) distribution of the open pool, with the main difference that no additional members will be joining after inception. This would be the case if an open pool were closed to new entrants at some point in the future or if a pool were created to wind up an existing defined benefit pension plan. This case is more conservative than that used in Section 4.2, as the membership is older, on average, in this illustration.

Table 3 reports the results associated with this alternative way of defining the initial membership of the closed pool. Again, the bulk of the changes are for longer horizons and no-go years. Indeed, the expected average benefit for no-go years is higher than in the base case (see Table 1). Yet the expected standard deviations of adjustments and benefits are also higher during these years. This increase in the risk measures is also felt in the hybrid measures: the aBaR and the CEC measures are worse for this type of initial age distribution than the results presented in Table 1, generally speaking. These changes are reasonable because the pool ages more quickly under this assumption, leading to more uncertainty in the benefit streams.

Figure 4.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR CLOSED AND OPEN POOLS WITH GROUP ADJUSTMENTS AND FOR DIFFERENT SETS OF INITIAL CONDITIONS.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both closed and open pools and for two different sets of initial conditions: pool inception in 2022 and the stationary economic scenario. We show the average (solid line) as well as 5th and 95th quantiles (dashed lines). We do so for two different types of pools. First, we consider a closed pool (in blue) with one thousand 65-year-old members; second, we assume an open pool (in green) where the pool's membership is assumed to be stationary (i.e., a membership consistent with a plan operating for a very long time). This figure relies on group adjustments of Section 2.2.

Table 5.

REWARD, RISK, AND HYBRID MEASURES FOR CLOSED AND OPEN POOLS FOR DIFFERENT SETS OF INITIAL CONDITIONS.

Measures	Pool inception in 2022				Stationary economic scenario			
	Group		Cohort		Group		Cohort	
	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool	Closed pool	Open pool
Expected average benefits (in $000s)								
Go-go years	77.0	77.0	77.0	77.0	83.0	83.0	83.0	83.0
Slow-go years	72.8	72.8	72.8	72.8	83.0	83.0	83.0	83.0
No-go years	75.1	73.8	75.1	74.1	84.1	83.6	84.1	83.8
Expected standard deviation of adjustments								
Go-go years	8.6%	8.6%	8.6%	8.6%	8.8%	8.8%	8.8%	8.8%
Slow-go years	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%
No-go years	12.1%	9.0%	12.1%	10.4%	12.0%	9.0%	12.0%	10.4%
Expected standard deviation of benefits (in $000s)								
Go-go years	8.9	8.9	8.9	8.9	10.5	10.5	10.5	10.5
Slow-go years	11.2	11.1	11.2	11.2	12.9	12.9	12.9	12.9
No-go years	14.7	11.5	14.7	12.9	16.3	12.8	16.3	14.3
Expected shortfall probability of adjustments, $C_k = 1.00$								
Go-go years	56%	56%	56%	56%	52%	52%	52%	52%
Slow-go years	52%	52%	52%	52%	52%	52%	52%	52%
No-go years	51%	51%	51%	52%	52%	52%	52%	53%
Expected shortfall probability of adjustments, $C_k = 0.95$								
Go-go years	32%	32%	32%	32%	30%	30%	30%	30%
Slow-go years	29%	29%	29%	29%	30%	30%	30%	30%
No-go years	32%	29%	32%	32%	33%	29%	33%	33%
Expected shortfall probability of benefits, $C_k = B_k(0)$								
Go-go years	69%	69%	69%	69%	57%	57%	57%	57%
Slow-go years	71%	71%	71%	71%	63%	63%	63%	63%
No-go years	69%	70%	69%	70%	65%	66%	65%	66%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$								
Go-go years	16%	16%	16%	16%	15%	15%	15%	15%
Slow-go years	42%	42%	42%	42%	38%	38%	38%	38%
No-go years	51%	52%	51%	52%	49%	49%	49%	49%
Minimum benefit at risk (in $000s)								
5-year horizon	34.2	34.2	34.2	34.3	34.8	34.8	34.8	34.8
10-year horizon	40.6	40.6	40.6	40.6	41.9	41.7	41.9	41.7
Average benefit at risk (in $000s)								
20-year horizon	19.8	19.7	19.8	19.7	27.7	27.7	27.7	27.7
30-year horizon	20.8	20.8	20.8	20.8	29.0	29.0	29.0	29.0
40-year horizon	21.9	21.7	21.9	21.8	30.0	29.5	30.0	29.7
Certainty equivalent consumption (in $000s)								
20-year horizon	61.8	61.8	61.8	61.9	64.0	64.0	64.0	64.0
30-year horizon	53.8	53.8	53.8	53.7	54.8	54.8	54.8	54.7
40-year horizon	45.2	46.2	45.2	43.7	46.0	46.6	46.0	44.9
50-year horizon	5.4	38.5	5.4	30.1	6.1	39.3	6.1	31.4

Notes: This table reports the various measures introduced in Section 3 for both closed and open pools and for group- and cohort-based adjustments. “Group” stands for group-based adjustments, and “cohort” for cohort-specific adjustments. We consider pools of 1,000 members at inception in this table. This table considers two sets of initial conditions: a case for which the pool inception is in 2022 and a case that starts at the stationary economic scenario.

4.5 IMPACT OF THE ASSET ALLOCATION

The base case of Section 4.2 assumed a 50% allocation to the stock index and 50% to the bond portfolio. To assess the importance of this assumption, we consider two additional allocation strategies:

1. 25% in the stock index and 75% in the bond portfolio.
2. 75% in the stock index and 25% in the bond portfolio.

Table 4 reports the various measures for these two robustness cases. Interestingly, changing the asset allocation does not change our earlier conclusions with respect to the design options: open pools have lower expected average benefits than closed pools but also smaller expected standard deviation measures. Again, the aBaR and CEC measures are overall better under the open pools, especially for longer horizons.

4.6 IMPACT OF INITIAL CONDITIONS

In the results above, we assumed the plan would start in 2023, when interest rates were higher than the assumed stationary level. In fact, some of these results could be a by-product of the initial conditions used in the simulations; initiating the pool at a different time could lead to different results. In this section, we consider a pool beginning its operation in 2022 and a pool starting in a scenario consistent with long-run economic and financial outputs; we call this latter scenario the *stationary economic scenario*. Figure 4 mirrors Figure 2 for the two sets of initial conditions; the left panel shows the case for which the pool inception is in 2022, and the right panel the stationary economic scenario case. The general behaviour of these two panels (and of Figure 2) are similar: both open and closed pools yield similar benefit streams, except for older members. After age 95, closed pools tend to be riskier—especially in the right tail of the benefit distribution. The average benefit is also higher in this case. Interestingly, the average benefit stream was increasing when starting in 2023—when interest rates were high. It is decreasing at first if we begin in 2022 because interest rates were lower than their steady states in this scenario. If we begin our simulation with the stationary economic scenario, the average benefit is level.

Table 5 reports the various measures for the two different sets of initial conditions. As shown in Figure 4 already, the expected average benefit decreases at first when starting in 2022, and it is level in the first half of retirement when the stationary economic scenario is used as the starting point in our scenario generation.

The other measures are qualitatively similar to those presented in Table 1: the numbers differ, but their interpretation and relative rankings remain the same. We again find that open pools tend to behave better in the long run when looking at their standard deviations, aBaR measures, and CECs for long horizons. The expected average in no-go years is somewhat smaller for open pools when compared to closed ones.

To conclude this section, there are some risk–reward trade-offs when designing closed and open pools, especially over longer horizons. Generally speaking, closed pools lead to more reward and risk during no-go years from the pool's perspective. When looking at hybrid measures like aBaR and the CEC, open pools perform better: after accounting for both reward and risk simultaneously, closed pools have the potential for very dire benefit scenarios due to less mortality pooling potential later on. Open pools benefit from new members participating in mortality pooling, which reduces the uncertainty of benefit streams in later years.

We use open pools in our simulations and tests in the remainder of this report.

Section 5: Hurdle Rate Policy

This section examines different hurdle rate policies. Specifically, as an alternative to the fixed hurdle rate policy used in Section 4, we consider a variable hurdle rate that adjusts to changes in future portfolio return expectations.

5.1 SETUP AND ASSUMPTIONS

We use the open pool setup of the last section as the starting point of this study, along with all the main assumptions introduced in Section 4.1. Two different hurdle rate policies are considered:

1. Fixed hurdle rate policy: The fixed hurdle rate policy relies on the long-run return of the portfolio determined at inception; that is, $h^{[t]} = h$ for all t. This is the assumption used in Section 4.
2. Variable hurdle rate policy: This policy uses a time-dependent hurdle rate in Equations (4) and (6); that is, the hurdle rate changes from one year to the next to account for changing economic conditions and as a function of the maturity of the payments.

In essence, the variable hurdle rate policy reprices the benefit each year to reflect the pool operator's then-current expectations of future portfolio returns and takes the effect of this repricing into account when determining benefit adjustments.

Modelling the variable hurdle rate policy is much more computationally complex than the base case since the annuity prices need to be recalculated along each simulation path, requiring nested simulations. The resulting adjustment factors capture not only the investment and mortality experience of the pool but also the impact of changes in the annuity prices. In our notation, this latter impact appears in the mortality adjustment factor. Specifically,

$$\mathrm{MEA}_k(t) = \begin{cases} \dfrac{\sum_{j\,\in\,\mathcal{L}_{t-1}^+} B_j(t-1)\, p_{x_j+t-1}^{[t-1]}\, \ddot{a}_{x_j,t}^{[t-1]}}{\sum_{j\,\in\,\mathcal{L}_t} B_j(t-1)\, \ddot{a}_{x_j,t}^{[t]}} & \text{for group-based adjustments} \\ \dfrac{\ddot{a}_{x_k,t}^{[t-1]}}{\ddot{a}_{x_k,t}^{[t]}} \left(p_{x_k+t-1}^{[t-1]} + q_{x_k+t-1}^{[t-1]}\, G(t)\right) & \text{for cohort-based adjustments} \end{cases},$$

where $\ddot{a}_{x_j,t}^{[t-1]}$ and $\ddot{a}_{x_j,t}^{[t]}$ are computed using information about the distribution of future portfolio returns as of time $t-1$ and t, respectively.

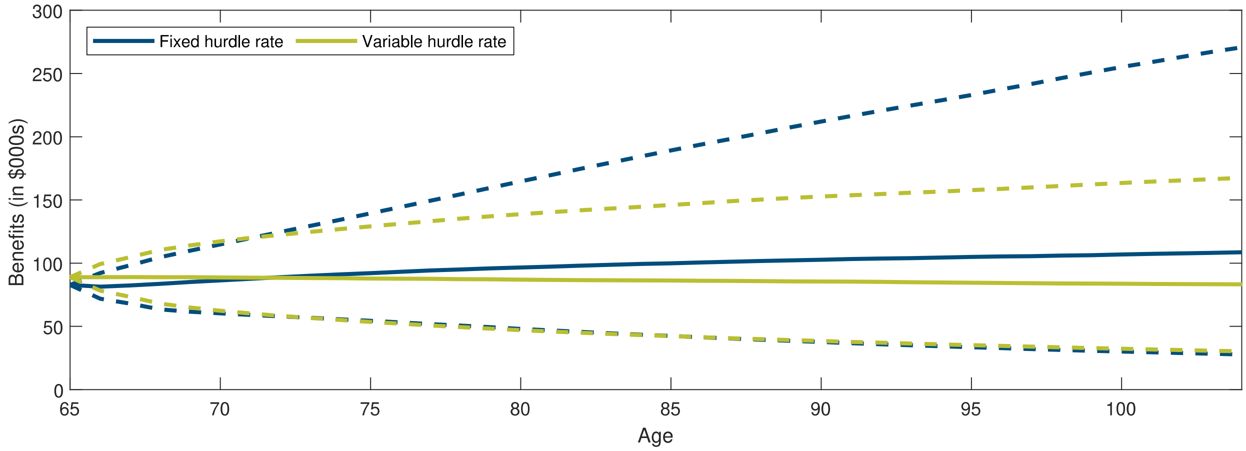
5.2 RESULTS ON HURDLE RATE POLICY

We assume the pool begins in 2023. We generate benefit streams using group- and cohort-based adjustments for both fixed and variable hurdle rate policies using 25,000 scenarios. We do so by assuming an open pool of 1,000 members—similar to Section 4.

5.2.1 EXPECTED AVERAGE BENEFITS

Figure 5 shows the funnel of doubt for the annual benefits using the fixed and variable hurdle rate policies; this figure relies on the group-based adjustments (note that cohort-based adjustments yield a similar figure; this is omitted for conciseness).

Figure 5.
ANNUAL BENEFIT FUNNEL OF DOUBT FOR FIXED AND VARIABLE HURDLE RATE POLICIES WITH GROUP ADJUSTMENTS.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both fixed and variable hurdle rate policies. We show the average (solid line) as well as 5th and 95th quantiles (dashed lines). We do so for two different types of pools. First, we consider a fixed hurdle rate (in blue); second, we assume a variable hurdle rate (in green). In both cases, the pool's membership is assumed to be stationary (i.e., a membership consistent with a plan operating for a very long time), with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2. The initial annual benefit of a 5-year-old member at inception is $88,654 for the variable hurdle rate policy and $82,758 for the fixed hurdle rate policy.

When the hurdle rate is fixed, the average benefit increases over the first 10–20 years, similar to Section 4. Again, this is due to interest rates being higher than their steady states at the beginning of 2023 and expected to return to their steady states in the long run, on average. With a variable hurdle rate, the average benefit stays roughly the same, as the hurdle rate is adjusted to capture changing economic conditions.

Table 6 reports the different reward, risk, and hybrid measures for fixed and variable hurdle rate policies as well as for group- and cohort-based adjustment factors. The expected average benefits over the go-go, slow-go, and no-go years are consistent with the results of Figure 5. For the fixed hurdle rate case, the average benefits are increasing at first—a by-product of the high interest rate at the beginning of 2023 when compared to those observed over the last thirty years; for the variable hurdle rate policy, on the other hand, we have relatively steady average benefits over time.

5.2.2 EXPECTED STANDARD DEVIATIONS OF ADJUSTMENT AND BENEFITS

Not only is the general profile of the average benefit stream different between fixed and variable hurdle rate policies but so is the benefit uncertainty. Indeed, after the first ten years or so, the benefit uncertainty associated with the variable hurdle rate becomes lower than that of the fixed hurdle rate, leading to a narrower funnel of doubt for the variable hurdle rate policy. This result implies that the benefits tend to be less uncertain when using the variable hurdle rate; this also leads to a reduced standard deviation of benefits, on average.

In addition, the expected standard deviations of adjustments are generally slightly lower with a variable hurdle rate when compared to the fixed hurdle rate policy in Table 6. For instance, using a group-based adjustment, it is about 8.0% for the variable hurdle rate, whereas it is 9.0% for the fixed rate.

An explanation for this behaviour can be provided by considering a specific realized scenario to help us visualize how benefits are updated. Figure 6 shows an example of annual benefits, annual (group-based) adjustment factors, as well as the separate mortality and investment adjustment factors for the fixed hurdle rate policy (in blue) and the variable hurdle rate policy (in green). We complement Figure 6 with Figure 7, documenting the various stock index, bond, and portfolio returns for the realized path used to generate Figure 6.

Table 6.
REWARD, RISK, AND HYBRID MEASURES FOR FIXED AND VARIABLE HURDLE RATE POLICIES.

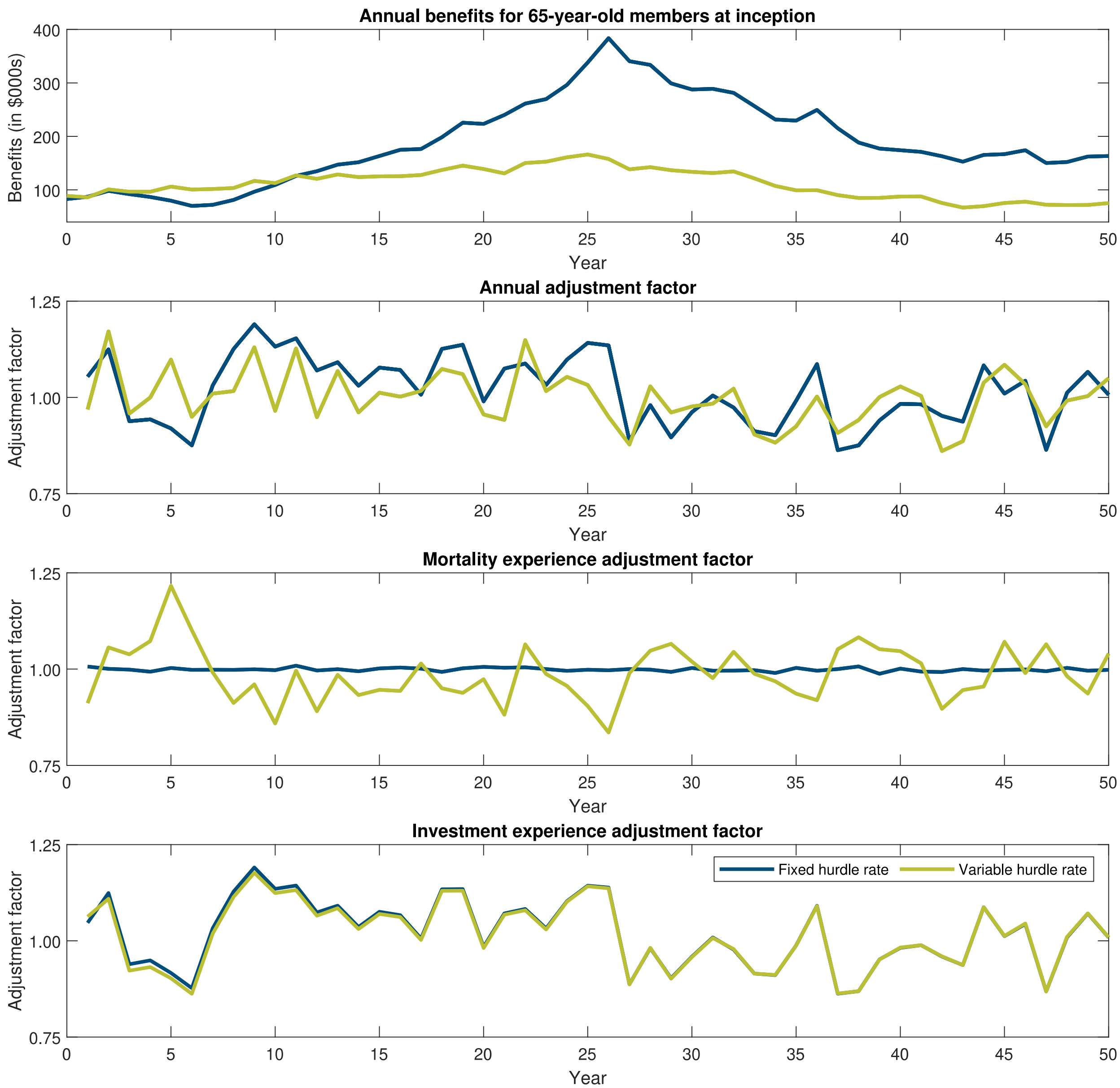
Measures	Group		Cohort	
	Fixed hurdle rate	Variable hurdle rate	Fixed hurdle rate	Variable hurdle rate
Expected average benefits (in $000s)				
Go-go years	86.9	88.6	86.9	88.6
Slow-go years	98.4	86.5	98.4	86.7
No-go years	106.1	84.1	106.4	84.5
Expected standard deviation of adjustments				
Go-go years	9.3%	8.1%	9.3%	8.3%
Slow-go years	9.1%	8.0%	9.2%	7.8%
No-go years	9.0%	8.0%	10.4%	9.3%
Expected standard deviation of benefits (in $000s)				
Go-go years	10.9	9.0	10.9	9.2
Slow-go years	15.6	10.5	15.6	10.4
No-go years	16.7	10.3	18.5	12.1
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	45%	50%	45%	50%
Slow-go years	48%	50%	48%	50%
No-go years	50%	50%	51%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	25%	25%	25%	26%
Slow-go years	27%	25%	27%	24%
No-go years	28%	25%	31%	29%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	44%	52%	44%	52%
Slow-go years	41%	59%	41%	58%
No-go years	47%	63%	47%	63%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$				
Go-go years	7%	10%	7%	10%
Slow-go years	18%	27%	18%	28%
No-go years	29%	39%	29%	39%
Minimum benefit at risk (in $000s)				
5-year horizon	29.4	32.7	29.4	33.2
10-year horizon	32.2	37.5	32.2	38.1
Average benefit at risk (in $000s)				
20-year horizon	26.8	23.6	26.8	24.2
30-year horizon	30.1	23.4	30.0	23.8
40-year horizon	32.2	22.4	32.4	22.8
Certainty equivalent consumption (in $000s)				
20-year horizon	74.8	74.2	74.8	73.9
30-year horizon	68.1	68.1	68.0	67.8
40-year horizon	61.1	62.7	59.4	61.3
50-year horizon	53.7	57.5	46.0	51.1

Notes: This table reports the various measures introduced in Section 3 for an open pool with fixed and variable hurdle rate policies combined with group- and cohort-based adjustments. We consider pools of 1,000 members at inception in this table. “Group” stands for group-based adjustments, and “cohort” for cohort-specific adjustments.

From Figure 6, both hurdle rate policies lead to different benefit streams (see the top panel of the figure). This is expected, as these two policies rely on different update rules. The adjustments also differ between fixed and variable hurdle rates (see the second panel of Figure 6); although they seem correlated and evolve similarly, a close inspection reveals sizeable differences. In both cases, these adjustments are related to the portfolio returns of Figure 7 (see bottom panel): in the fixed hurdle rate case, the adjustments are more closely associated with these returns, whereas they are more dissimilar in the variable hurdle rate case.

Figure 6.

EXAMPLE OF A REALIZED PATH OF ANNUAL BENEFIT, ANNUAL ADJUSTMENT FACTORS, AS WELL AS MORTALITY AND INVESTMENT ADJUSTMENT FACTORS FOR FIXED AND VARIABLE HURDLE RATE.

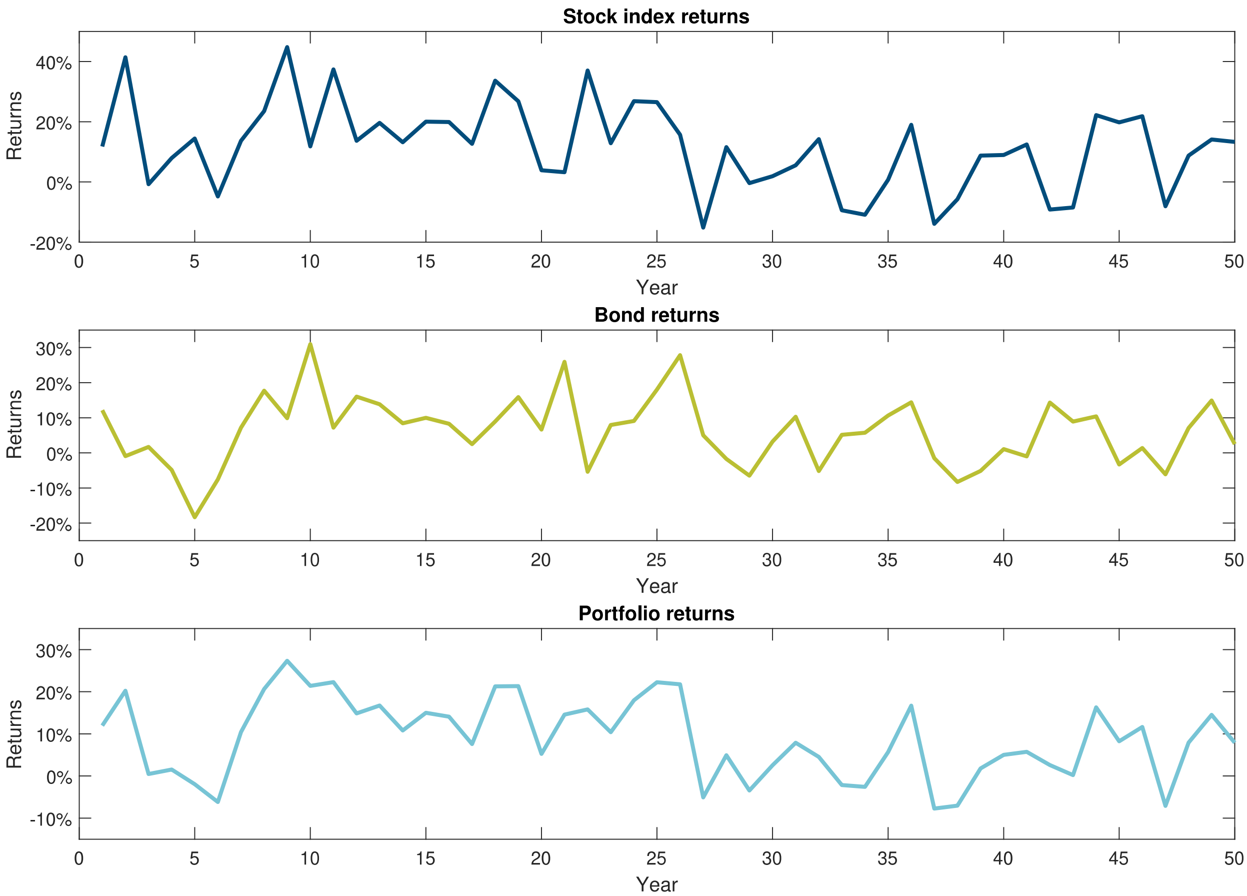


Notes: This figure reports an example of the annual benefits of a 65-year-old member at inception, annual adjustment factors, as well as the mortality experience and investment experience adjustment factors. We do so for two different types of pools. First, we consider a fixed hurdle rate (in blue); second, we assume a variable hurdle rate (in green). For the variable hurdle rate, the mortality experience adjustment factor includes the impact of changes in the hurdle rate. In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

When breaking down the adjustment factors into a mortality experience component and an investment experience component, we witness a very different behaviour between fixed and variable hurdle rate policies. Even though the investment experience adjustment factors are similar in both cases (see the bottom panel of Figure 6), the mortality experience adjustment factors are at odds (see the third panel of the same figure). With the fixed hurdle rate policy, the mortality adjustment is almost constant and close to one: the hurdle rate is not changing, so the uncertainty comes only from the number of decedents and survivors in this case. With the variable hurdle rate, the adjustments are volatile: as the hurdle rate is changing over time, the annuity prices are changing as well, adding more volatility to the mortality experience adjustment factor.[33]

Figure 7.

EXAMPLE OF A REALIZED PATH OF ANNUAL RETURNS ON THE STOCK INDEX, BOND, AND PORTFOLIO.



Notes: This figure reports an example of a realized path of the stock index, bond, and portfolio returns. The portfolio is based on a 50–50 split between the index and the bond portfolio.

Because the adjustment factor is the product of both investment and mortality experience factors, the adjustment variance should be higher when using the variable hurdle rate policy, as both parts are moving—not only the investment experience adjustment factor, as in the fixed hurdle rate case. Specifically, in our experiment above, the expected standard deviation of the mortality experience factor is about 6.8% in the variable case; it is almost nil when using the fixed hurdle rate assumption. However, we observe a strong negative correlation (about –51%) between the two components of the adjustment factor when using a variable hurdle rate policy, thus significantly reducing the

[33] In the context of the variable hurdle rate, we acknowledge that the mortality experience adjustment calculated in accordance with Section 5.1 is not only related to mortality—it also includes the impact of changes in the hurdle rate assumption on the annuity prices. Rather than introducing an alternative definition of the MEA for this purpose, we have decided to keep the terminology introduced in Section 2 to maintain consistency with other studies in the literature.

total standard deviation of adjustments in this case. In fact, the negative correlation between IEA and MEA in the case of the variable hurdle rate policy is so strong that it reduces the risk that the adjustment factor becomes largely different than one.

The rationale above explains why the expected standard deviation of adjustments is lower when using the variable hurdle rate policy. This lower expected standard deviation of adjustments indeed spills over into the expected standard deviation of benefits: having smaller benefit changes in absolute value makes the benefit standard deviation lower as well.

Intuitively, the variable hurdle rate policy allows for a better match between the cost of the future benefits and the asset value from one year to another when compared to the fixed hurdle rate policy. This ultimately reduces interest rate risk.

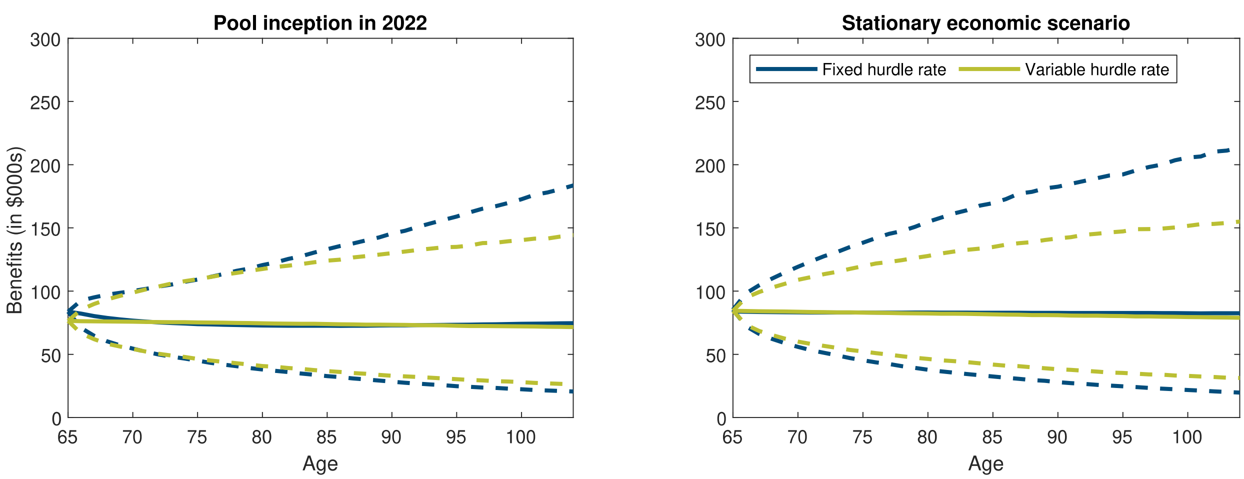
5.2.3 OTHER MEASURES

Table 6 also reports expected shortfall probabilities, benefit at risk, and certainty equivalent consumption measures.

The expected shortfall probabilities of adjustments are similar for both fixed and variable hurdle rate policies. The expected shortfall probabilities of benefits, on the other hand, are different: they tend to be smaller for the fixed hurdle rate cases than for the variable cases. This difference is due to the sizeable benefit increases granted under the fixed hurdle rate policy in the first 10–20 years after 2023. This short-term difference also drives the mBaR measures: they tend to be better for the fixed hurdle rate policies.

Figure 8.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR AN OPEN POOL WITH FIXED AND VARIABLE HURDLE RATE POLICIES FOR DIFFERENT SETS OF INITIAL CONDITIONS.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both fixed and variable hurdle rate policies and for two different sets of initial conditions: pool inception in 2022 and a stationary economic scenario. We show the average (solid line) as well as 5th and 95th quantiles (dashed lines). First, we consider a fixed hurdle rate (in blue); second, we assume a variable hurdle rate (in green). In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Unlike the mBaR, the aBaR measures are smaller (i.e., better) for the variable hurdle rate policy. Over the long run, members are expected to lose less with the variable hurdle rate policy than with the fixed one, on average. This is mirrored in the CEC measures, which tend to be better with the variable hurdle rate over 40- and 50-year horizons.

5.3 IMPACT OF INITIAL CONDITIONS

Similar to Section 4.6, we again assess the impact of changing the set of initial conditions. We consider, once more, a pool beginning its operation in 2022 and a pool starting at the stationary economic scenario. Figure 8 reports the funnels of doubt for these two sets of initial conditions.

Table 7.

REWARD, RISK, AND HYBRID MEASURES FOR FIXED AND VARIABLE HURDLE RATE POLICIES FOR DIFFERENT SETS OF INITIAL CONDITIONS.

	Pool inception in 2022				Stationary economic scenario			
	Group		Cohort		Group		Cohort	
Measures	Fixed hurdle rate	Variable hurdle rate	Fixed hurdle rate	Variable hurdle rate	Fixed hurdle rate	Variable hurdle rate	Fixed hurdle rate	Variable hurdle rate
Expected average benefits (in $000s)								
Go-go years	77.0	75.7	77.0	75.7	83.0	83.6	83.0	83.7
Slow-go years	72.8	74.2	72.8	74.3	83.0	81.9	83.0	82.0
No-go years	73.8	72.3	74.1	72.7	83.6	79.8	83.8	80.0
Expected standard deviation of adjustments								
Go-go years	8.6%	7.8%	8.6%	8.0%	8.8%	7.6%	8.8%	7.7%
Slow-go years	9.0%	8.0%	9.0%	7.8%	9.0%	7.6%	9.0%	7.5%
No-go years	9.0%	8.0%	10.4%	9.4%	9.0%	7.6%	10.4%	8.9%
Expected standard deviation of benefits (in $000s)								
Go-go years	8.9	7.6	8.9	7.8	10.5	8.0	10.5	8.2
Slow-go years	11.1	9.0	11.2	8.8	12.9	9.6	12.9	9.4
No-go years	11.5	8.8	12.9	10.5	12.8	9.3	14.3	11.0
Expected shortfall probability of adjustments, $C_k = 1.00$								
Go-go years	56%	50%	56%	50%	52%	50%	52%	51%
Slow-go years	52%	50%	52%	50%	52%	50%	52%	50%
No-go years	51%	50%	52%	51%	52%	50%	53%	51%
Expected shortfall probability of adjustments, $C_k = 0.95$								
Go-go years	32%	25%	32%	26%	30%	24%	30%	25%
Slow-go years	29%	25%	29%	24%	30%	24%	30%	23%
No-go years	29%	25%	32%	29%	29%	24%	33%	28%
Expected shortfall probability of benefits, $C_k = B_k(0)$								
Go-go years	69%	54%	69%	54%	57%	54%	57%	54%
Slow-go years	71%	59%	71%	59%	63%	59%	63%	59%
No-go years	70%	63%	70%	63%	66%	63%	66%	63%
Expected shortfall probability of benefits, $C_k = 0.75\ B_k(0)$								
Go-go years	16%	9%	16%	9%	15%	9%	15%	9%
Slow-go years	42%	27%	42%	27%	38%	26%	38%	26%
No-go years	52%	39%	52%	39%	49%	38%	49%	38%
Minimum benefit at risk (in $000s)								
5-year horizon	34.2	27.4	34.3	27.8	34.8	29.8	34.8	30.0
10-year horizon	40.6	32.0	40.6	32.3	41.7	34.4	41.7	34.7
Average benefit at risk (in $000s)								
20-year horizon	19.7	19.7	19.7	20.0	27.7	21.1	27.7	21.4
30-year horizon	20.8	19.4	20.8	19.6	29.0	20.7	29.0	21.0
40-year horizon	21.7	18.6	21.8	18.9	29.5	19.9	29.7	20.2
Certainty equivalent consumption (in $000s)								
20-year horizon	61.8	64.2	61.9	64.0	64.0	72.0	64.0	71.9
30-year horizon	53.8	58.8	53.7	58.7	54.8	66.9	54.7	66.8
40-year horizon	46.2	53.6	43.7	52.3	46.6	62.3	44.9	61.5
50-year horizon	38.5	48.6	30.1	42.3	39.3	58.2	31.4	54.3

Notes: This table reports the various measures introduced in Section 3 for an open pool with fixed and variable hurdle rate policies combined with group- and cohort-based adjustments. We consider pools of 1,000 members at inception in this table. "Group" stands for group-based adjustments, and "cohort" for cohort-specific adjustments.

When the pool starts operating in 2022, the average benefit pattern is the opposite of that observed in Figure 5 for 2023: the fixed hurdle rate initial benefit starts higher than the variable hurdle rate benefit, but the former decreases over time as the interest rates are expected to increase. There is more uncertainty when relying on the fixed hurdle rate policy, and the uncertainty differential grows with time; that is, the fixed hurdle rate funnel of doubt becomes much wider than that of the variable hurdle rate.

Both average benefit streams are somewhat constant and identical for the stationary economic scenario. The funnel of doubt for the fixed hurdle rate policy is wider than that of the variable hurdle rate, hinting that the benefits and adjustments under the fixed hurdle rate policy are more uncertain.

Table 7 reports the reward, risk, and hybrid measures for these different sets of initial conditions. Again, we consider group- and cohort-based adjustments as well as fixed and variable hurdle rate policies.

For each given set of initial conditions, the expected average benefit is similar across the adjustment factors and the hurdle rate policies. Note that it is slightly decreasing when the starting point of our simulation is 2022, which is consistent with the fact that interest rates were low at the beginning of 2022.

The expected standard deviation of adjustments and benefits share comparable patterns to those observed in the base case of Section 5.2: the standard deviations of adjustments and benefits are lower for the variable hurdle rate.

In contrast to the base case, the expected shortfall probabilities of benefits are lower when using the variable hurdle rate for both 2022 and the stationary economic scenario. For instance, the probability of having lower benefits than the starting benefit is 3% higher when using the fixed hurdle rate in the stationary economic scenario.

These lesser probabilities indicate a thinner left tail in the case of the variable hurdle rate. The thinner tails also impact the mBaR and aBaR measures: they are systematically lower when considering the variable hurdle rate policy for both the 2022 scenario and the stationary economic scenario. The CEC measures are also better in the two cases considered in this robustness case when using the variable hurdle rate.

So, here, the set of initial conditions clearly matters: for low-interest rate environments and those environments that are close to *typical* conditions in the economy, we expect the variable hurdle rate policy to perform better. Yet when interest rates are high, the fixed hurdle rate policy can lead to higher benefits on average. There could therefore be a trade-off in some cases, and pool designers need to understand it.

5.4 INFLATIONARY INCREASES BUILT INTO THE DESIGN

Sometimes, pool designers want to target an increasing benefit stream to maintain members' purchasing power. This strategy is typically achieved by artificially reducing the hurdle rate so that the expected investment experience adjustment factors are higher than one, on average. While future benefit increases are uncertain, this strategy makes them more likely than benefit cuts. There is, of course, a cost to these benefit increases: the benefit at inception is reduced to be consistent with the lower hurdle rate. This means that the initial benefits presented in this section are lower than those in earlier sections to be able to afford the inflationary increases built into the design.[34]

In this section, we compare two different hurdle rate policies that target a benefit stream that is level in real terms (i.e., once inflation has been accounted for). Specifically, we define the two designs as follows:

[34] Recall that reducing the hurdle rate increases the price of the annuity, which ultimately decreases the benefit at inception; see Equation (3) for more details.

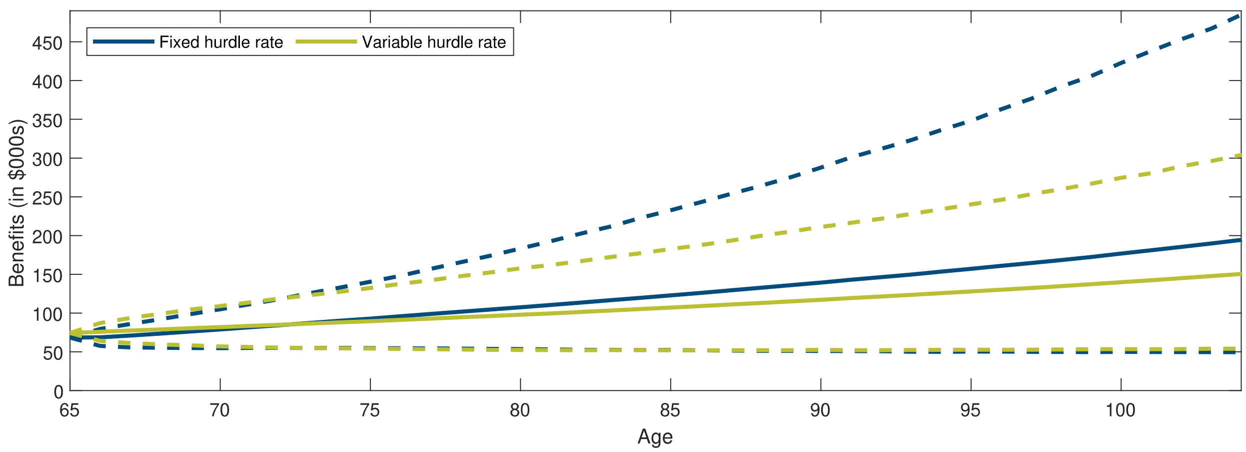
1. Fixed hurdle rate policy: We set the hurdle rate to a constant at inception, as in Section 4. However, this constant is now equal to the median long-term rate of return on the portfolio, minus the long-run inflation rate (i.e., about 2% in our setting). This should allow for a 2% benefit increase per annum, on average.
2. Variable hurdle rate policy: The second design considers a variable hurdle rate that is based on the distribution of future portfolio returns, minus the distribution of future inflation rates, evolving with economic conditions. In other words, we consider future portfolio return expectations on an inflation-adjusted basis.

The first design is easy to implement, as the annuity prices rely on a fixed hurdle rate that is not path dependent. The second design, on the other hand, requires nested simulations to determine the annuity prices along each path.

Figure 9 shows the funnel of doubt of benefits based on the two hurdle rate policies above. The fixed hurdle rate policy's initial benefit is lower than that obtained with the variable hurdle rate. This should not come as a surprise, as the 2023 expected portfolio return is higher than its long-run value—a by-product of the interest rates observed at the beginning of 2023—leading to a higher initial hurdle rate in the variable case.

Figure 9.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR FIXED AND VARIABLE HURDLE RATE POLICIES WITH GROUP ADJUSTMENTS AND FOR POOLS WITH INFLATIONARY INCREASES BUILT INTO THE DESIGN.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both fixed and variable hurdle rate policies. We show the average (solid line) as well as 5th and 95th quantiles (dotted lines). We do so for two different types of pools. First, we consider a fixed hurdle rate that targets inflationary adjustments (in blue); second, we assume a variable hurdle rate that equals the average real return on the portfolio (in green). In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Since the hurdle rate is lower than the (inflation-adjusted) average returns in the first 10–20 years in the fixed hurdle rate case, the benefits grow faster than their variable hurdle rate counterparts. This result is similar to the behaviour observed in Figure 5, where benefits for the fixed hurdle rate were growing during the first few years. Nonetheless, the average rate of increase of the benefits is similar for both designs after the first 20 years, and both policies are effective in producing an expected stream of payments that increases by roughly 2% per year.

Consistent with Figures 5 and 8, the benefit uncertainty tends to be larger when the hurdle rate is fixed, with a wider funnel of doubt in that case. The expected standard deviations of benefits reported in Table 8 are aligned with this observation: they are systematically lower for the variable hurdle rate policy, and this for both group- and cohort-based adjustments.

Table 8.

REWARD, RISK, AND HYBRID MEASURES FOR FIXED AND VARIABLE HURDLE RATE POLICIES FOR A POOL WITH INFLATIONARY INCREASES BUILT INTO THE DESIGN.

Measures	Group		Cohort	
	Fixed hurdle rate	Variable hurdle rate	Fixed hurdle rate	Variable hurdle rate
Expected average benefits (in $000s)				
Go-go years	80.4	82.7	80.4	82.8
Slow-go years	117.0	103.6	117.0	103.9
No-go years	169.5	135.4	170.1	136.1
Expected standard deviation of adjustments				
Go-go years	9.5%	8.4%	9.5%	8.7%
Slow-go years	9.3%	8.2%	9.3%	8.0%
No-go years	9.2%	8.2%	10.4%	9.3%
Expected standard deviation of benefits (in $000s)				
Go-go years	12.2	9.6	12.2	9.9
Slow-go years	22.0	14.9	22.0	14.7
No-go years	30.7	19.5	33.3	21.8
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	36%	40%	36%	41%
Slow-go years	39%	40%	39%	39%
No-go years	41%	40%	43%	42%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	19%	19%	19%	20%
Slow-go years	20%	18%	20%	18%
No-go years	21%	18%	24%	22%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	27%	32%	27%	33%
Slow-go years	15%	23%	15%	23%
No-go years	13%	16%	13%	16%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$				
Go-go years	3%	4%	3%	4%
Slow-go years	4%	7%	4%	7%
No-go years	5%	6%	6%	6%
Minimum benefit at risk (in $000s)				
5-year horizon	21.1	23.5	21.1	24.3
10-year horizon	20.7	24.6	20.7	25.4
Average benefit at risk (in $000s)				
20-year horizon	29.0	25.9	29.0	26.5
30-year horizon	37.3	29.1	37.4	29.5
40-year horizon	46.1	31.6	46.5	32.3
Certainty equivalent consumption (in $000s)				
20-year horizon	75.3	76.1	75.3	75.7
30-year horizon	76.1	77.0	76.1	76.6
40-year horizon	76.4	77.9	75.7	77.2
50-year horizon	76.2	78.7	72.8	76.1

Notes: This table reports the various measures introduced in Section 3 for both fixed and variable hurdle rate policies with targeted inflationary increases and for group- and cohort-based adjustments. We consider pools of 1,000 members at inception in this table. "Group" stands for group-based adjustments, and "cohort" for cohort-specific adjustments.

Again, the lower standard deviation of benefits aligns with lower standard deviation of adjustments when using the variable hurdle rate. As in Section 5.2, the impact of changing interest rates on portfolio returns is offset by the impact of updating the hurdle rate, reducing the uncertainty of the total adjustments.

Table 8 also reports many other measures. While the shortfall probabilities of adjustments are similar for both designs, the shortfall probabilities of benefits differ: they tend to be more significant for the variable hurdle rate policy. As already mentioned in Section 5.3, this is a by-product of the initial conditions. Starting in 2022 or at the stationary economic scenario makes these probabilities lower for the variable hurdle rate policy when compared to the fixed hurdle rate policy.

The mBaR measures are more considerable when using the variable hurdle rate policy. This should not be surprising, as changing the hurdle rate can potentially create more uncertainty in the short term.

However, the benefits should become more stable under the variable hurdle rate policy match between the portfolio returns and changes in the cost of benefits. We indeed find that the aBaR measures are lower for the variable hurdle rate policy, suggesting that the risk–reward trade-off between these two designs favours the variable hurdle rate policy. This result is also consistent with the CECs, especially those over the long term. For instance, for group-based adjustments, the 50-year horizon CEC is $78,700 when using the variable hurdle rate and group adjustments; it is only $76,200 for the fixed hurdle rate case.

5.5 A COMMENT ON BEHAVIOURAL FINANCE AND FAIRNESS

To conclude this section, we note some risk and reward trade-offs when using different hurdle rate policies:

1. As mentioned above, the fixed hurdle can lead to higher average benefits in some cases; this is driven by the economic conditions at inception. If the current rates are higher than their steady state, then we expect the benefit to increase in the short term. On the other hand, if the rates are lower than their long-run values, we should expect benefit cuts in the short term.
2. Generally speaking, the variable hurdle rate policy leads to less uncertainty in both the benefit stream and the year-to-year variability in the benefits. The fixed hurdle rate policy, on the other hand, yields more benefit and adjustment uncertainty.
3. When combining risk and reward measures using the benefit at risk and the CEC measures, the variable hurdle rate policy appears superior to the fixed hurdle rate policy for all economic environments considered in this report.

Interestingly, variable hurdle rates might also lead to additional fairness among generations, as members are expected to receive level benefits, on average, that are not impacted by short-term fluctuations in economic conditions.

A potential detractor from the variable hurdle rate policy is that variable rates need to be determined by the pool operator—via a model or otherwise—which involves a degree of subjectivity. The fixed hurdle rate policy appears to be more objective, on the other hand, as the rates are set in advance.

Fixed hurdle rates are also simpler to understand for members—it might be very hard for a financially illiterate member to grasp variable hurdle rates and why they are changing from one year to another. Additionally, significantly positive returns that would have brought considerable benefit increases with the fixed hurdle rate policy might only lead to negligible benefit increases with the variable hurdle rate policy (see, e.g., Figure 6). Communicating these results will prove to be a challenge for pool operators.

For convenience, we will rely on the fixed hurdle rate policy in our simulations and tests for the remainder of this report. This choice is motivated by the fact that the fixed hurdle rate policy is simpler to understand for members and to implement, making it a better option from a behavioural finance perspective. Moreover, for the remainder of this report, we will only consider the group-based adjustments, as results do not drastically differ between group- and cohort-based adjustments when using an open group (except for older members).

Section 6: Delayed Recognition of Gains and Losses

This section considers the potential for delayed recognition of gains and losses via three different procedures inspired by current practices in lifetime pension pools. We begin the section by introducing each procedure. Then another measure that supplements those presented in Section 3 is introduced; it accounts for a new dimension that was not relevant before—*shortfall risk*, or the risk of the cost of benefits exceeding assets. We apply the adjustment smoothing methods to open lifetime pension pools with group-based adjustments, as mentioned above. Similar to Section 5.4, we again investigate pools that offer inflation protection to members, but now in the light of delayed recognition. The section concludes by discussing fairness, intergenerational equity, and the ability to attract new members.

6.1 PROCEDURES FOR DELAYED RECOGNITION OF GAINS AND LOSSES

This report investigates three explicit methods for delaying the recognition of gains and losses, in addition to the base case (i.e., immediate recognition):[35]

1. Staggered adjustments: A method that *spreads* out the adjustments arising from each year's experience over a fixed number of years so that the actual adjustment applied to time-t benefits is a function of the realized adjustments over the last n years.
2. Adjustment corridor: A method that puts caps and floors on the actual adjustment applied to time-t benefits. For instance, if the time-t adjustment $\alpha(t)$ is lower than $\alpha_{\min}$, we apply a change of $\alpha_{\min}$ to the benefits and delay the difference to later years. If, on the other hand, the adjustment $\alpha(t)$ is larger than $\alpha_{\max}$, we similarly apply $\alpha_{\max}$ and delay the rest of the increases.
3. Hurdle rate adjustments: A method that allows for some variation in the hurdle rate used to determine the cost of the pool's benefits. We constrain the hurdle rate to a range between some maximum and minimum so that it does not fall too far out of alignment with the current economic environment. Specifically, if the hurdle rate needed to match the pool's assets with the cost of benefits is above $h_{\max}$, we cap the hurdle rate at this value and modify the time-t benefits consistently. On the other hand, if the rate needed is below $h_{\min}$, we set the hurdle rate to $h_{\min}$ and again modify the time-t benefits consistently.

When implementing these three procedures, we make sure not to apply past experience to the benefits of new entrants in the pool, since doing so would be unfair.[36]

6.1.1 STAGGERED ADJUSTMENTS

Under this method, the actual adjustments (i.e., those applied to the benefits) are computed by combining the realized adjustments over the last n years in order to smooth them. Here, the realized adjustments are calculated as in Section 2.

Because the adjustment factors multiply the benefits, we rely on a geometric average of the last n adjustment factors so that

[35] We consider methods for payout smoothing that are facilitated inside and by the pool (i.e., explicit), which introduce complexities and might create some complications. Note that each member could perform payout smoothing *outside* the pool or on their own (i.e., implicit). For instance, a member could own two accounts: one pooled (illiquid) account and one non-pooled (liquid) account. Then if the pool account were to fail to deliver decent benefits, the member could withdraw from their non-pooled account to make up for the shortfall.

[36] One could argue that new members would have been most likely impacted by the same investment experience during their accumulation phase and should not be penalized twice for potential losses.

$$\dot{\alpha}(t) = \left(\prod_{s=0}^{n-1} \alpha(t-s)\right)^{\frac{1}{n}},$$

where $\dot{\alpha}(t)$ denotes the actual adjustments applied to the time-t benefits under the staggered adjustment method, and n is called the *window size*. Note that the pools introduced in Section 2 can be recovered by setting n to one.

This method only delays the inevitable: if the realized adjustment at time t is 0.9 and n is set to 5, then the benefits will be reduced by about 2% every year for the next five years. This allows members to adjust their expenses gradually over time.

Another benefit of this approach is that the geometric average used to find $\dot{\alpha}(t)$ leads to less volatility than the realized adjustments, making the benefit stream more stable over time.[37] There is a cost to this advantage, however: the pool's assets are not perfectly aligned with the cost of benefits anymore, introducing shortfall risk. The size of this risk depends on the window size: the longer the window, the greater the shortfall risk.[38] There is, therefore, a trade-off between benefit stability and the potential for shortfall—a common compromise in the pension literature.

One reason for this shortfall is timing mismatch. For instance, let us assume the time-t realized return is very low, which negatively impacts the assets. As this smaller asset value is reinvested, the future returns in dollars are also expected to be smaller. Yet the pool has not fully adjusted the benefits it is paying its members—the benefits were only partially revised because of the geometric averaging—which decreases the pool assets at a faster rate than assumed by the realized adjustments.

There are also some timing mismatches related to the impact of deaths. If a member dies before benefits are fully adjusted, the surviving members' mortality gains need to be reduced to compensate for benefit losses not accounted for yet in the case of a negative adjustment. This can also create some potential for shortfalls in the short term. This shortfall can impact fairness and intergenerational equity. It might also affect the pool's ability to attract new members. The last subsection of Section 6 will discuss these implications.

Our implementation of the staggered adjustments method is not unique; it was selected for its ease of execution. It also allows for adjustments that are easy to communicate to members, as the actual adjustments are directly tied to the realized ones. However, as noted above, it creates a potential for shortfall due to the mismatch between the asset value and the cost of the benefits.

Other implementations might reduce the risk of shortfall. It would in fact be possible to remove all shortfall risk: specifically, one could structure staggered benefit adjustments in future years in such a way that a member's adjusted benefit stream would be actuarially equal to the value of assets currently available. This, unfortunately, may also imply that benefit cuts or increases could be larger in absolute terms—thus increasing the standard deviation of benefits.[39] Studying other staggered adjustment mechanisms is left for future research.

6.1.2 ADJUSTMENT CORRIDOR

For the adjustment corridor method, the actual adjustments are computed by capping and flooring the realized adjustments calculated in Section 2. Specifically, the actual adjustments are given by

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[37] It is well known that an average tends to be less volatile than the actual observations in a sample. Averaging in this case relies on the same principle.
[38] In the limit, as the window size tends to infinity, we obtain a fixed annuity with large shortfall risk and a high probability of bankruptcy (i.e., asset value reaching zero).
[39] It might also be difficult to communicate to members benefit cuts that appear larger than those implied by the no-smoothing method.

$$\ddot{\alpha}(t) = \begin{cases} \alpha_{\min} & \text{if } \alpha^*(t) < \alpha_{\min} \\ \alpha^*(t) & \text{if } \alpha_{\min} \leq \alpha^*(t) \leq \alpha_{\max} \\ \alpha_{\max} & \text{if } \alpha^*(t) > \alpha_{\max} \end{cases},$$

where $\ddot{\alpha}(t)$ denotes the actual adjustments applied to the time-t benefit under the adjustment corridor method, $\alpha_{\min}$ is the lower bound of the corridor, $\alpha_{\max}$ is the upper bound of the corridor, and $\alpha^*(t)$ is the time-t realized adjustment combined with all past adjustments that were not accounted for, such that

$$\alpha^*(t) = \alpha(t)\,\frac{\alpha^*(t-1)}{\ddot{\alpha}(t-1)}.$$

In other words, any (current and past but not realized) adjustment that is within $\alpha_{\min}$ and $\alpha_{\max}$ is applied to the benefits, whereas adjustments that are above $\alpha_{\max}$ or below $\alpha_{\min}$ are modified so that the actual adjustment stays in the corridor. The remainder that has not been accounted for at time t is pushed to the following year.

This procedure indeed limits the size of the adjustments, thus decreasing the volatility of the adjustments (and the expected shortfall probabilities of adjustments as a by-product).

Again, similar to the staggered adjustment method, the adjustment corridor methodology simply delays the inevitable: large benefit increases or cuts are postponed but not completely eliminated. This delay, nonetheless, creates a mismatch between the pool's assets and the cost of benefits, generating shortfall risk: the method implies a potential for shortfall and, ultimately, bankruptcy. This could also impact fairness, intergenerational equity, and the ability of the pool to attract new members; we will address these important issues at the end of this section.

Finally, again, this implementation is not unique. Another noteworthy and interesting adjustment corridor method could have considered cumulative adjustments instead of annual adjustments. We leave the study of such an adjustment corridor mechanism for future research.

6.1.3 HURDLE RATE ADJUSTMENTS

The last procedure we consider for delaying the recognition of experience gains and losses relies on changing the hurdle rate. This is a fundamentally different approach than the previous ones: rather than first determining the adjustment required to keep benefits affordable and then smoothing those adjustments in some way, this method attempts to absorb experience gains and losses by repricing the benefits so that benefit adjustments may not be required at all.

To illustrate this process, let us assume that the hurdle rate at inception $h^{[0]}$ is set to the long-run median of the portfolio returns (as in Section 4). Then, at time 1, the asset value becomes

$$\sum_{j\,\in\,\mathcal{L}_{0^+}} B_j(0)\left(\ddot{a}^{[0]}_{x_j,0} - 1\right)\exp(r^{\text{PF}}_1)\,, \tag{9}$$

assuming that $\ddot{a}^{[0]}_{x_j,0}$ is calculated using $h^{[0]}$. The cost of future benefits at that time is given by

$$\sum_{j\,\in\,\mathcal{L}_1} B_j(0)\,\ddot{a}^{[1]}_{x_j,1}, \tag{10}$$

which could be made equal to Equation (9) if the hurdle rate used to compute $\ddot{a}^{[1]}_{x_j,1}$ were selected appropriately. It is possible to solve for the *implied* hurdle rate by setting Equations (9) and (10) equal—this rate forces $\alpha(1)$ to be equal to 1, and $B_j(0)$ to be equal to $B_j(1)$. Without any bounds on the hurdle rate, this process could be applied each year to keep the benefits constant; that is, $B_j(t) = B_j(t-1)$ for all t.

Note that changing the hurdle rate in this way also changes the pattern of future expected benefits; for example, lowering the hurdle rate would make future investment experience gains more likely. However, if the implied hurdle rate described above were to be applied consistently each year, the cost of benefits would always be reset to the pool's then-current assets, and the benefit stream would stay constant indefinitely.

In practice, there may be limits to these hurdle rate adjustments: we assume in this design a minimum and maximum hurdle rate to keep the plan somewhat aligned with the current economic environment. These bounds are denoted by $h_{\min}$ and $h_{\max}$, respectively. The hurdle rate applicable at time t is then given by

$$h^{[t]} = \begin{cases} h_{\min} & \text{if } h^* < h_{\min} \\ h^* & \text{if } h_{\min} \le h^* \le h_{\max} \\ h_{\max} & \text{if } h^* > h_{\max} \end{cases},$$

where h^* is the implied hurdle rate that forces the cost of benefits associated with the most recent benefit level (set at time $t-1$) to be equal to the assets available at time t. Using this implied hurdle rate, we can now compute the actual benefit adjustment:

1. If the hurdle rate $h^{[t]}$ is between $h_{\min}$ and $h_{\max}$, then the adjustment is set to $\ddot{\alpha}(t) = 1$.
2. If the hurdle rate $h^{[t]}$ is $h_{\min}$ or $h_{\max}$, then the adjustment factor becomes

$$\ddot{\alpha}(t) = \left(\frac{\sum_{j \in \mathcal{L}_{t-1^+}} B_j(t-1) \left(\ddot{a}^{[t-1]}_{x_j,t-1} - 1 \right)}{\sum_{j \in \mathcal{L}_t} B_j(t-1)\, \ddot{a}^{[t]}_{x_j,t}} \right) \exp(r_t^{\mathrm{PF}}),$$

where $\ddot{a}^{[t]}_{x_j,t}$ is computed using $h^{[t]}$ above.

In the first case, when the hurdle rate falls between $h_{\min}$ and $h_{\max}$, the benefits do not change. Otherwise, when the hurdle rate reaches either $h_{\min}$ or $h_{\max}$, the adjustment is such that the cost of the updated future benefits (calculated using the constrained hurdle rate) is equal to the pool's assets.

This approach allows the pool to hold back investment and mortality gains to fund an implicit reserve that can be released later when experience is unfavourable, potentially shifting consumption from earlier years (and cohorts) to later ones. Conversely, it allows the current generations to borrow from the future to keep benefits constant in cases where they might have been reduced under the base case.

Unlike the other methods to delay recognition of the gains and losses, this procedure does not create any potential shortfall, since the assets always match the cost of benefits (at least as those costs are measured by the pool). There is, nonetheless, a potential for bankruptcy if the hurdle rate bounds are not conservative enough.[40]

6.2 AN ADDITIONAL MEASURE: RELATIVE SHORTFALL VALUE AT RISK

Some adjustment smoothing mechanisms explained above involve a potential for shortfall (i.e., the cost of future benefit payments, as determined by the pool, exceeding the assets accumulated to fund those payments) as well as

[40] Note that to apply this smoothing mechanism to cohort-based adjustments would require having a different hurdle rate for each cohort, as adjustments are cohort-dependent.

bankruptcy (i.e., the assets of the pool falling to zero). We introduce a new measure that captures shortfall risk to supplement those presented in Section 3.

Let us define the pool's liabilities at time t, denoted by $L(t)$, as the cost of future benefits based on the hurdle rate determined by the pool at time t. Let us further define the time-t relative shortfall as the difference between these liabilities and the pool's assets, divided by the assets; that is,

$$\mathcal{S}(t) = \frac{L(t) - A(t)}{A(t)}.$$

To understand the extent of the potential mismatch between the pool's liabilities and assets, we compute the value at risk of the relative shortfall distribution:

$$\mathrm{VaR}_p[\mathcal{S}(t)] = F_{\mathcal{S}(t)}^{-1}(p),$$

where $F_{\mathcal{S}(t)}^{-1}(p)$ is the quantile function of the (random) relative shortfall evaluated at p. Simply put, this measure gives us the maximum relative shortfall after excluding all worse outcomes whose combined probability is at most p.

We select 5-, 10-, and 20-year horizons for the calculations of the relative shortfall value at risk. The probability level p is set to 95% throughout our calculations.

It is important to recognize that the *liabilities* defined above are not true liabilities in an accounting sense. This is because a lifetime pension pool provides no guarantees about benefits levels and could indeed adjust benefits at any time, so its liability for accounting purposes is always equal to its assets. By contrast, $L(t)$ is a shorthand for the cost of future benefits as determined by the pool itself (based on its chosen hurdle rate and other assumptions about future contingencies), which from time to time may differ from its assets.[41] Similarly, the relative shortfall measures should not be interpreted as the risk of being *underfunded* but rather as an indication of the past performance of the pool and whether it will be attractive for potential future members to join.

Finally, we could have assessed bankruptcy risk instead of shortfall risk by looking at cases where the pool's assets reach zero. However, this should be rather infrequent given reasonable design features (i.e., window size, corridor width, and hurdle rate bounds), and this would not be very informative in the end. Therefore, we focus only on shortfall risk in this report.

6.3 RESULTS ON DELAYED RECOGNITION

6.3.1 STAGGERED ADJUSTMENTS

The first method—staggered adjustments—is applied to the open pool investigated above, still starting with 1,000 members. We use a fixed hurdle rate set at the long-run median return of the portfolio and consider only group-based adjustments. In this application, we select a five-year window, meaning that adjustments are smoothed out over five years.[42]

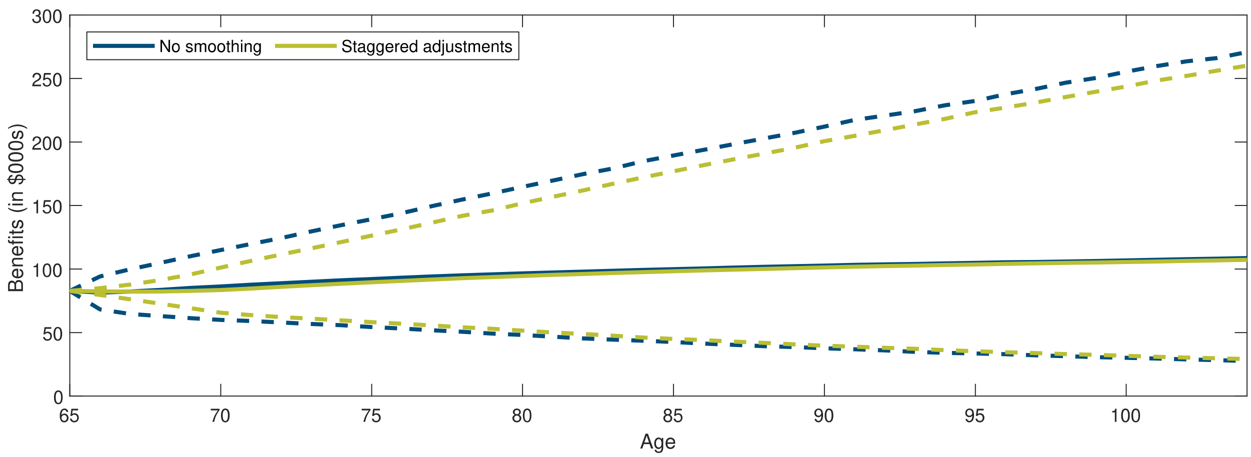
Figure 10 reports the funnel of doubt for the annual benefits of a 65-year-old member at inception for the staggered adjustment method and the base case (i.e., no smoothing). Overall, the average benefit under both assumptions is similar, albeit slightly smaller, if using the staggered adjustment method; this method also leads to thinner tails and

[41] As a result, there is no *objectively correct* value of $L(t)$: it is based on the subjective choices of the pool.
[42] Note that in this case (and throughout our report) new cohorts coming in are not impacted by past adjustments out of a concern for fairness.

less extreme benefits. This is a by-product of the adjustment smoothing that reduces the volatility of adjustments (i.e., about 4% for the staggered adjustment method and 9% for the base case). Table 9 reports similar information about the expected average and standard deviation of benefits for go-go, slow-go, and no-go years; overall, these numbers are consistent with Figure 10.

Figure 10.

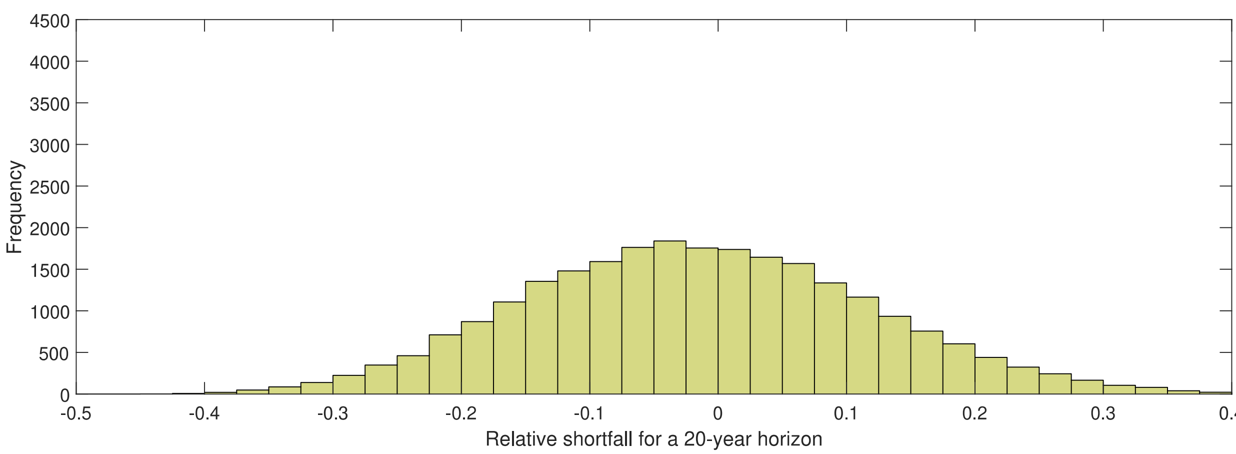
ANNUAL BENEFIT FUNNEL OF DOUBT FOR STAGGERED ADJUSTMENTS WITH AN OPEN POOL, GROUP ADJUSTMENTS, AND A FIXED HURDLE RATE.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both the no-smoothing base case and the staggered adjustment method. We show the average (solid line) as well as 5th and 95th quantiles (dotted lines). We do so for two different types of pools. First, we consider a pool with no adjustment factor smoothing (like that introduced in Section 2, in blue); second, we apply the staggered adjustment method of Section 6.1.1 that spreads out adjustments over a five-year period (in green). In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Figure 11.

RELATIVE SHORTFALL FOR A 20-YEAR HORIZON USING STAGGERED ADJUSTMENTS WITH AN OPEN POOL, GROUP ADJUSTMENTS, AND A FIXED HURDLE RATE.



Notes: This figure reports a histogram of the distribution of the random relative shortfall over a 20-year horizon for the staggered adjustment method. We apply the staggered adjustment method of Section 6.1.1 that smooths out adjustments over a five-year period. The pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Table 9.

REWARD, RISK, HYBRID, AND SHORTFALL MEASURES FOR DELAYED RECOGNITION OF GAINS AND LOSSES.

Measures	No smoothing	Group Staggered adjustments	 Adjustment corridor	 Hurdle rate adjustments
Expected average benefits (in $000s)				
Go-go years	86.9	84.9	85.1	85.0
Slow-go years	98.4	96.6	94.6	103.8
No-go years	106.1	104.8	102.2	124.1
Expected standard deviation of adjustments				
Go-go years	9.3%	3.0%	3.9%	4.6%
Slow-go years	9.1%	3.6%	3.7%	5.6%
No-go years	9.0%	3.6%	3.7%	5.5%
Expected standard deviation of benefits (in $000s)				
Go-go years	10.9	7.3	7.0	8.4
Slow-go years	15.6	13.0	11.5	17.5
No-go years	16.7	13.9	12.4	20.6
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	45%	45%	45%	15%
Slow-go years	48%	48%	48%	20%
No-go years	50%	53%	52%	25%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	25%	7%	25%	8%
Slow-go years	27%	12%	28%	12%
No-go years	28%	14%	32%	14%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	44%	47%	46%	34%
Slow-go years	41%	41%	41%	41%
No-go years	47%	46%	46%	45%
Expected shortfall probability of benefits, $C_k = 0.75\, B_k(0)$				
Go-go years	7%	3%	2%	6%
Slow-go years	18%	16%	15%	21%
No-go years	29%	28%	27%	32%
Minimum benefit at risk (in $000s)				
5-year horizon	29.4	20.7	18.7	24.9
10-year horizon	32.2	25.8	24.0	30.3
Average benefit at risk (in $000s)				
20-year horizon	26.8	22.5	21.2	29.5
30-year horizon	30.1	26.7	24.8	37.8
40-year horizon	32.2	29.4	27.2	44.3
Certainty equivalent consumption (in $000s)				
20-year horizon	74.8	77.5	79.4	72.3
30-year horizon	68.1	71.1	74.6	60.7
40-year horizon	61.1	64.1	68.4	49.3
50-year horizon	53.7	56.7	61.6	38.0
Relative shortfall value at risk				
5-year horizon	0.0%	10.3%	7.2%	0.0%
10-year horizon	0.0%	12.6%	9.8%	0.0%
20-year horizon	0.0%	21.1%	16.7%	0.0%

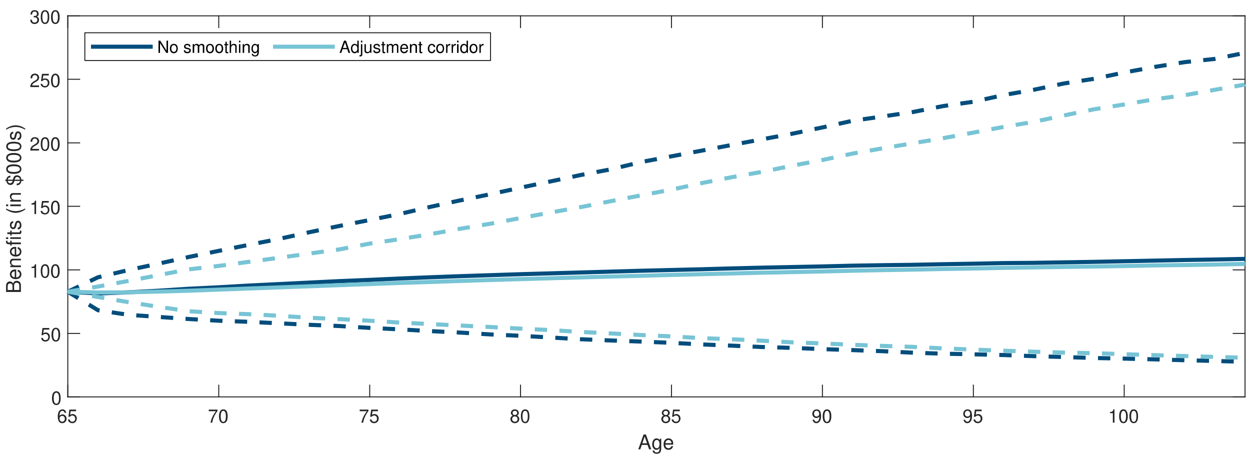
Notes: This table reports the various measures introduced in Sections 3 and 6 for open pools with group-based adjustments. We consider pools of 1,000 members at inception in this table. "Group" stands for group-based adjustments.

Extreme downward adjustments are significantly less likely when using the staggered adjustment method, as can be seen from the expected shortfall probabilities of adjustments with a threshold C_k of 0.95 (see Table 9). By contrast, the staggered adjustment method can lead to more benefit cuts that are smaller in size (see the increase in the expected shortfall probability of adjustments with a threshold of 1.00). Overall, the shortfall probabilities of benefits are positively impacted: they are slightly lower with the staggered adjustment method than in the base case.

The minimum and average benefit at risk measures are systematically better when we apply staggered adjustments, with the highest gains for the minimum BaRs. This result is mainly explained by having less extreme benefit scenarios when smoothing, as reported in Figure 10 already. These less extreme benefit scenarios also improve the CECs for all horizons, with increases of about 5% on average.

The only measure that is negatively impacted by the delayed recognition of gains and losses is the relative shortfall value at risk: these are systematically positive for the staggered adjustment method. By contrast, there is no shortfall risk under the base case: as benefits are adjusted directly when there is no smoothing, there is no potential for mismatches between the pool's assets and cost of benefits. With staggered adjustments, however, this becomes a reality. Consistent with Figure 11, the 95% value at risk of the relative shortfall at the end of 20 years is about 21%, meaning that in one of out 20 scenarios, the pool assets would need to be increased by at least 21% for the pool to have enough money to pay for future benefits.

Figure 12.
ANNUAL BENEFIT FUNNEL OF DOUBT FOR THE ADJUSTMENT CORRIDOR METHOD WITH AN OPEN POOL, GROUP ADJUSTMENTS, AND A FIXED HURDLE RATE.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both the no-smoothing base case and the adjustment corridor method. We show the average (solid line) as well as 5th and 95th quantiles (dotted lines). We do so for two different types of pools. First, we consider a pool with no adjustment factor smoothing (like that introduced in Section 2, in blue); second, we apply the adjustment corridor method of Section 6.1.2 that adds caps and floors on adjustment factors of 0.95 and 1.05, respectively (in light blue). In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

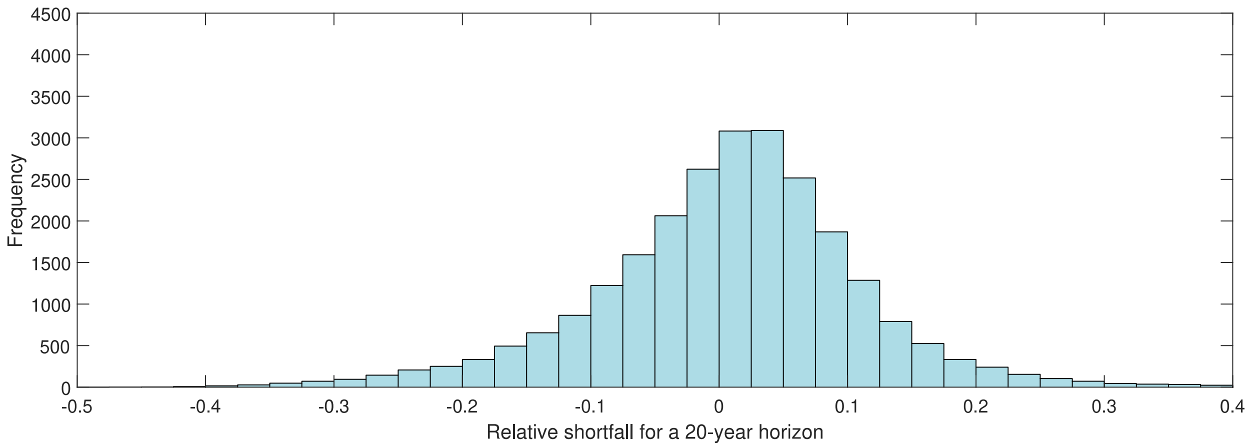
6.3.2 ADJUSTMENT CORRIDORS

We consider next an adjustment corridor with a minimum adjustment $\alpha_{\min}$ of 0.95 and a maximum adjustment $\alpha_{\max}$ of 1.05; this means that all adjustments above 1.05 or below 0.95 are capped or floored, respectively.

Figure 12 reports the annual benefit funnel of doubt for a 65-year-old member at inception based on the adjustment corridor method as well as the base case. Overall, the average benefit with the adjustment corridor is slightly below that of the base case—about $3,000 lower. The future benefit distribution tends to be narrower when capping and flooring the adjustment factors than that observed without smoothing: there is less benefit downside risk but also less upside potential.

Figure 13.

RELATIVE SHORTFALL FOR A 20-YEAR HORIZON USING THE ADJUSTMENT CORRIDOR METHOD WITH AN OPEN POOL, GROUP ADJUSTMENTS, AND A FIXED HURDLE RATE.

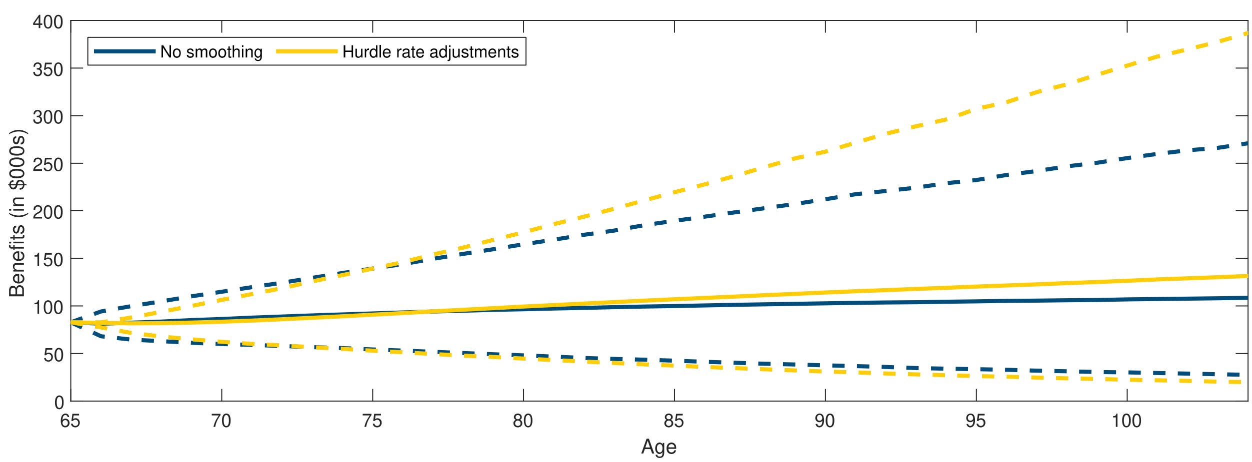


Notes: This figure reports a histogram of the distribution of the random relative shortfall over a 20-year horizon for the adjustment corridor method. We apply the adjustment corridor method of Section 6.1.2 that adds caps and floors on adjustment factors of 0.95 and 1.05, respectively. The pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

This is consistent with the expected standard deviation of benefits reported in Table 9: they are systematically lower for the adjustment corridor method when compared to the base case in each of the three periods considered (i.e., go-go, slow-go, and no-go years). As expected, the expected standard deviation of adjustments is also lower with caps and floors on the adjustments.

Figure 14.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR THE HURDLE RATE ADJUSTMENT METHOD WITH AN OPEN POOL, GROUP ADJUSTMENTS, AND A FIXED HURDLE RATE.



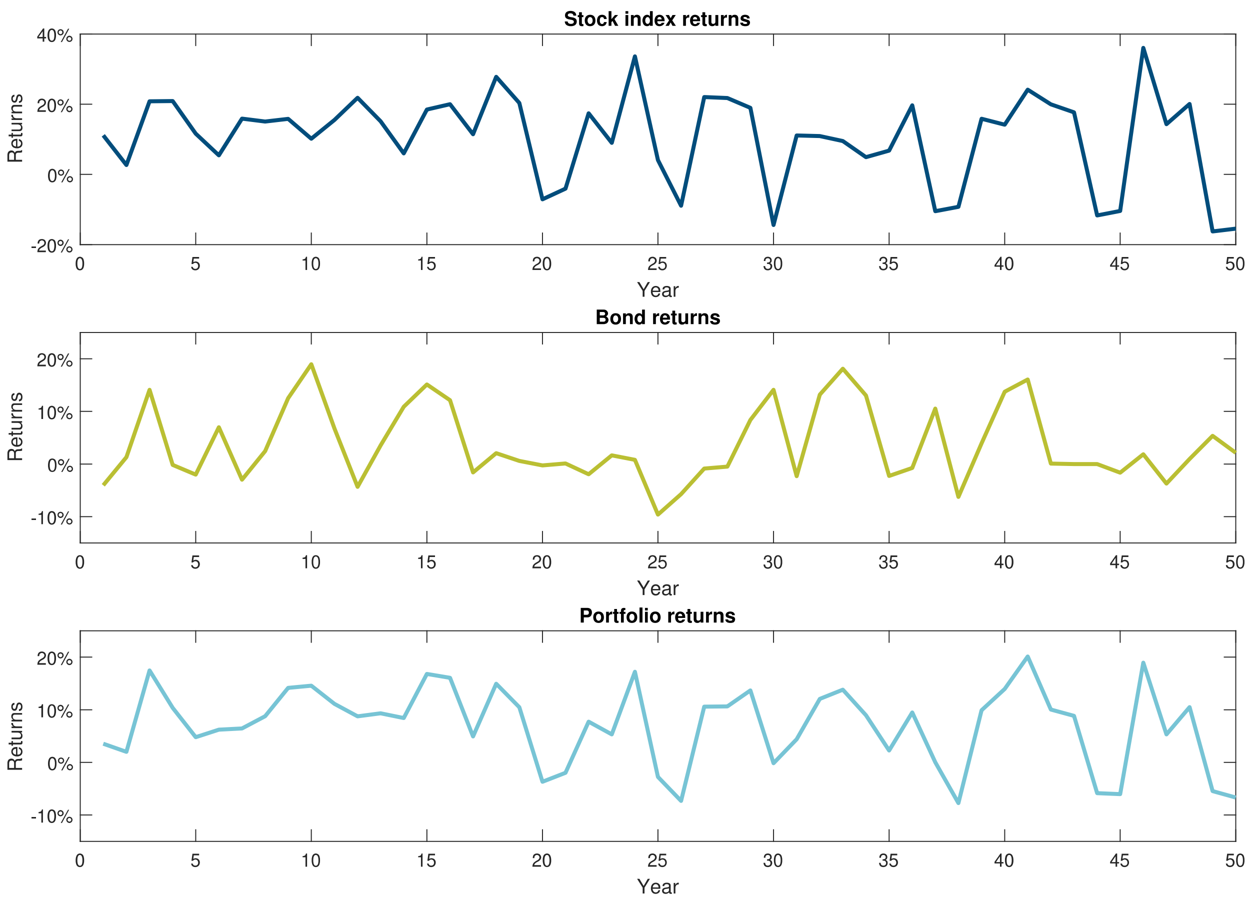
Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for both the no-smoothing base case and the hurdle rate adjustment method. We show the average (solid line) as well as 5th and 95th quantiles (dotted lines). We do so for two different types of pools. First, we consider a pool with no adjustment factor smoothing (like that introduced in Section 2, in blue); second, we apply the hurdle rate adjustment method of Section 6.1.3 that allows the hurdle rate to be updated. In both cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

The adjustment corridor method also marginally reduces the shortfall probabilities of benefits when compared to the base case. By contrast, the method yields larger adjustment shortfall probabilities: they tend to be 1% to 4% higher in the medium to long term than in the base case when the threshold is set at 0.95. This conclusion is intuitive, as our floor is set to 0.95, and any additional adjustments not applied in the current year are carried forward, increasing the potential for benefit cuts in the future.

Note that although benefit cuts of exactly 5% are more likely to occur, the sizes of benefit cuts in general are smaller under the adjustment corridor method than under the base case, since the latter can far exceed 5%. As a result, the benefit at risk measures are systematically better when using the adjustment corridor method due to the decrease in the likelihood of extreme cuts. The CECs are also higher than those of the base case, again pointing to a slightly better risk–reward profile when allowing for adjustment caps and floors.

There is a cost, however, to adding this corridor—an increase in the shortfall risk. The 20-year relative shortfall value at risk is about 17% for the adjustment corridor method. This means that after 20 years of operation, the pool is projected to be missing at least 17% of its assets to cover the cost of future payouts based on the current benefit level in one out of 20 scenarios. Figure 13 shows the full distribution of the relative shortfall, for which most of the mass is close to zero—a good sign.

Figure 15.
EXAMPLE OF A REALIZED PATH OF ANNUAL RETURNS ON THE BOND, STOCK INDEX, AND PORTFOLIO.



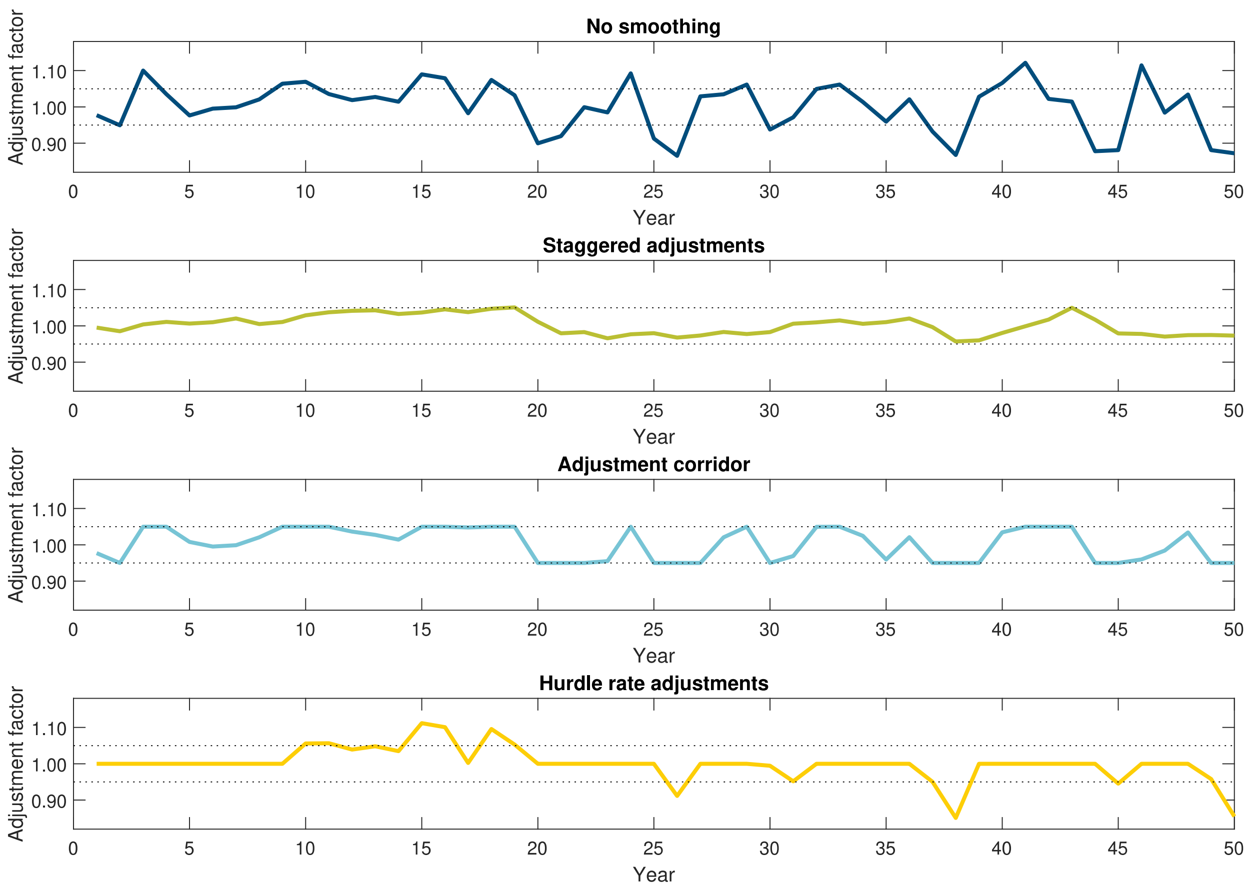
Notes: This figure reports an example of a realized path of the stock index, bond, and portfolio returns. The portfolio is based on a 50–50 split between the index and the bond portfolio.

When compared to the staggered adjustment method, the relative shortfall value at risk (as well as the whole distribution of Figures 11 and 13, to some extent) looks better when using the adjustment corridor method. This is also true for almost all measures in Table 9, except for the standard deviation of adjustments and shortfall probabilities of adjustments.

6.3.3 HURDLE RATE ADJUSTMENTS

The hurdle rate adjustment method is the last method we consider for delaying the recognition of gains and losses. In this application, we select $h_{\min}$ to be 2% lower than the initial hurdle rate (set to the median long-run portfolio return) and $h_{\max}$ to be 2% higher than the initial hurdle rate.

Figure 16.
EXAMPLE OF A REALIZED PATH OF ANNUAL ADJUSTMENT FACTORS USING VARIOUS METHODS TO DELAY GAINS AND LOSSES.



Notes: This figure reports an example of a realized path of adjustment factors for the base case (no smoothing), the staggered adjustment method, the adjustment corridor method, and the hurdle rate adjustment method. In all cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2. The dashed line corresponds to α_{min} and $\alpha_{\max}$, set at 0.95 and 1.05, respectively.

Figure 14 reports the funnel of doubt for the annual benefits of a 65-year-old member at inception for the hurdle rate adjustment method as well as the base case. In the first ten years, the benefits and their uncertainty tend to be smaller, on average, for the hurdle rate adjustment method. This is mainly explained by the hurdle rate staying between $h_{\min}$ and $h_{\max}$ in most paths during this period, making the benefit streams less variable. After the first ten years, however, the behaviour of benefit streams differs: the average benefits under the hurdle rate adjustment method increase and the funnel of doubt becomes wider with significantly more upside. This can be attributed to a compounding effect for gains (or losses) when the hurdle rate is at $h_{\min}$ or $h_{\max}$. Specifically, when the hurdle rate is at $h_{\min}$, the pool is ready to start giving away benefit increases. These increases tend to be higher and more frequent than those offered by other alternate plans because the hurdle rate is lower than the expected portfolio return. On the other hand, if the hurdle rate is at $h_{\max}$, the pool has delayed benefit cuts for as long as it could and has to start decreasing benefits. However, because the hurdle rate is higher now, it becomes more likely for the portfolio returns to fall short of it, which leads to larger and more frequent cuts, generally speaking. In other words, the hurdle rate adjustment method magnifies gains (losses) when the pool is in a good (bad) state.

Because the hurdle rate is changed in accordance with the financial situation of the pool, the standard deviation of adjustments tends to be lower than that of the base plan. Yet the benefit standard deviation is higher because of the *compounding* effect of extreme hurdle rates.

The shortfall probabilities of adjustments are better with this smoothing rule, a by-product of the no-action corridor. Yet when benefits are adjusted, the adjustment size tends to be larger than those observed with other types of delayed recognition techniques, especially after the first 15–20 years. Consequently, while the short-term minimum benefit at risk measures are better than those in the base case, the longer-term average benefit at risk measures are significantly worse than those of the base case (and worse than any other smoothing mechanisms). This is also true for the CECs, which are systematically and significantly lower than those of other methodologies; for instance, over a 50-year horizon, the hurdle rate adjustment method yields a CEC of 38.0 compared to a CEC of 53.7 in the base case.

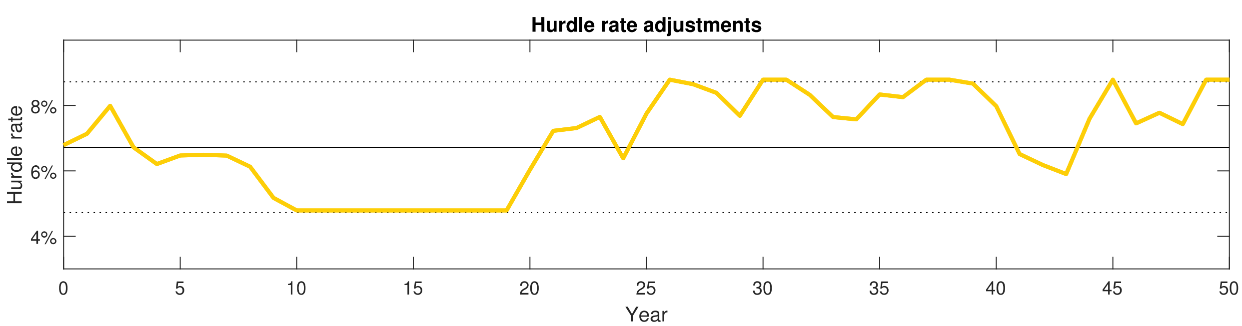
As mentioned earlier, the relative shortfall value at risk for the hurdle rate adjustment method is nil because the cost of future benefits (as determined by the pool) is always equal to the pool's assets.

6.3.4 SIMILARITIES AND DIFFERENCES BETWEEN THE METHODS

We investigate results for a specific economic and mortality scenario to understand and illustrate the similarities and differences between the different methods for delaying recognition. This is similar to the analysis in Section 5.2.

Figure 17.

EXAMPLE OF A REALIZED PATH OF HURDLE RATE FOR THE HURDLE RATE ADJUSTMENT METHOD.



Notes: This figure reports an example of a realized path of hurdle rate for the hurdle rate adjustment method of Section 6.1.3. The pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Figure 15 shows the realized annual returns on the stock index, bond, and portfolio for the specific case under study. In this case, the bond and stock returns hover around their long-run average, with some extremely bad outcomes throughout the years.

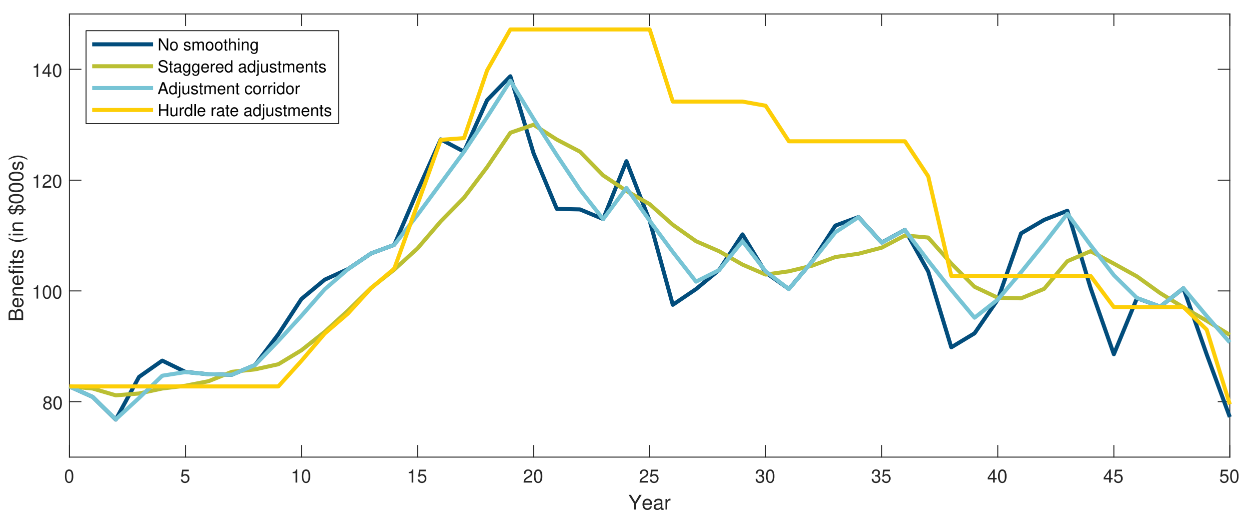
Based on this scenario, we can obtain adjustment factors for each of the four methods investigated in this section: the base case without any smoothing, the staggered adjustment method, the adjustment corridor, and the hurdle rate adjustment method. Figure 16 discloses the four series of adjustment factors.

Not surprisingly, the base case yields changes that are quite volatile. The adjustment factors could be higher than 1.05 or lower than 0.95: an adjustment of more than 5% happens 12 times, and an adjustment of less than –5% happens 12 times. This improves under the staggered adjustment and adjustment corridor methods: both methods yield adjustments that stay most of the time between these bounds. The staggered adjustment method tends to yield smoother adjustments; this result is consistent with the standard deviation of adjustments in Table 9.

The hurdle rate adjustment method yields long periods of stability without any benefit adjustments—this is when the hurdle rate is between $h_{\min}$ and $h_{\max}$ (see Figure 17). As soon as the rate reaches the lower or upper bound, however, benefits are adjusted. Positive adjustments—if justified by positive investment and mortality experience—are applied to the benefits when the hurdle rate is set to $h_{\min}$ (see, e.g., years 10 to 19 in Figures 16 and 17). Negative adjustments can occur when the hurdle rate is set to $h_{\max}$ (see, e.g., years 30 to 31 and 37 to 38 in Figures 16 and 17).

Figure 18 reports the benefit stream for this same scenario. The base case without any smoothing displays some erratic behaviour, as expected, mainly driven by the volatile portfolio returns (see Figure 15). The benefits paid under the staggered adjustment and adjustment corridor methods stay close to the base case, while displaying a smoother pattern. This is also expected, as these methods eliminate extremes by either averaging them or truncating them, respectively.

Figure 18.
EXAMPLE OF A REALIZED PATH OF ANNUAL BENEFIT FOR 65-YEAR-OLD MEMBERS AT INCEPTION.



Notes: This figure reports an example of a realized path of annual benefits of a 65-year-old member at inception for the base case (no smoothing), the staggered adjustment method, the adjustment corridor method, and the hurdle rate adjustment method. In all cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

The story is quite different when looking at the benefits under the hurdle rate adjustment method: the benefits stay rather constant at first when the hurdle rate is being adjusted. After the hurdle rate reaches its minimum in year 10, the benefit stream is positively adjusted because the pool can afford it. These benefit increases are higher than those obtained under the other methods because a lower hurdle rate makes the same portfolio returns appear better than otherwise (i.e., the difference between the realized return and the hurdle rate is higher when the hurdle rate is low). Then, starting from year 20, returns get lower, generally speaking (see Figure 15); after that point, the hurdle rate tends to be higher, and benefits need to be cut to keep up with poor financial results. Indeed, the benefits are negatively adjusted on multiple occasions after year 20 (i.e., in years 26, 30, 31, 37, 38, 45, 49, and 50). This happens at a time when the hurdle rate is high, however, making the adjustment worse than that observed with other mechanisms. This illustrates key features of the hurdle rate adjustment method:

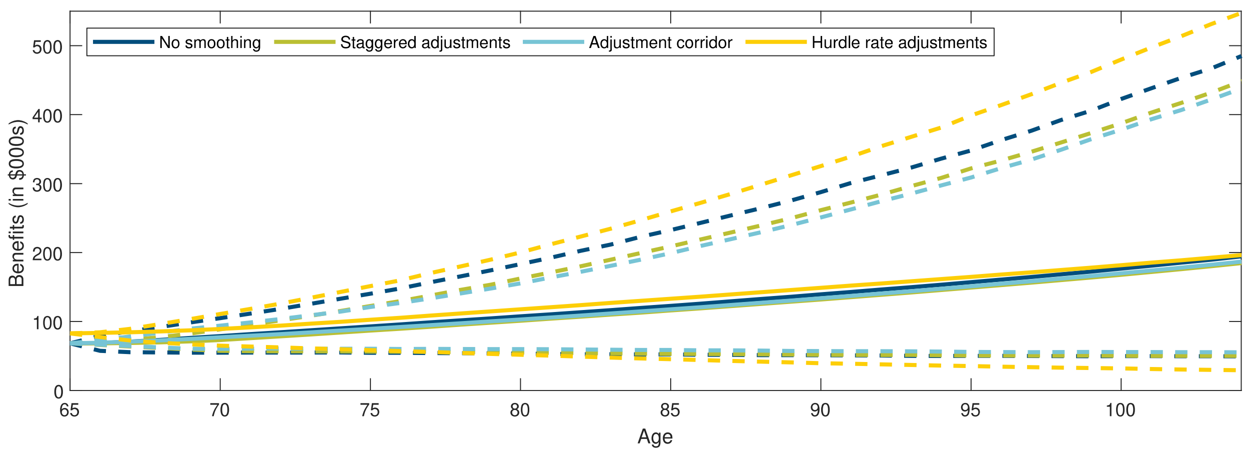
1. Low hurdle rates store capacity to withstand unfavourable experience and magnify favourable experience in future years.
2. With a hurdle rate at the top of the allowable range, the pool's benefits are at greater risk of being reduced, and rapidly so, since the pool has no capacity to absorb unfavourable experience and, worse yet, any such experience is magnified.

6.4 INFLATIONARY INCREASES BUILT INTO THE DESIGN

Akin to Section 5.4, we investigate pools for which benefits are expected to increase over time, but we do so now in the light of delayed recognition of gains and losses. In other words, we apply the three procedures explained above to a pool whose benefits should increase by an average of 2% per annum; this number is aligned with long-term inflation expectations in Canada. Again, recall that these increases are targeted but not guaranteed; they are only given if the pool can afford to do so. We target these increases by decreasing the hurdle rate used in the previous subsections by about 2%, making the adjustment factors 2% larger, on average.

Figure 19.

ANNUAL BENEFIT FUNNEL OF DOUBT FOR THE THREE METHODS FOR DELAYED RECOGNITION OF GAINS AND LOSSES WITH AN OPEN POOL, GROUP ADJUSTMENTS, A FIXED HURDLE RATE, AND INFLATIONARY INCREASES BUILT INTO THE DESIGN.



Notes: This figure reports funnels of doubt for the annual benefits of a 65-year-old member at inception for the base case (no smoothing), the staggered adjustment method, the adjustment corridor method, and the hurdle rate adjustment method (see Section 6.1 for more details). We show the average (solid line) as well as 5th and 95th quantiles (dotted lines). In all cases, the pool's membership is assumed to be open and stationary, with 1,000 members at inception. This figure relies on group adjustments presented in Section 2.2.

Specifically, we investigate the following four cases:

1. <u>No smoothing</u>: The adjustments are applied without smoothing, as given in Section 2—without any modifications.
2. <u>Staggered adjustments</u>: The adjustment factors are averaged for a period of five years. Note that they will tend to be higher than those obtained in the previous subsections because the hurdle rate is lower here, yielding adjustments that tend to be larger, on average.
3. <u>Adjustment corridor</u>: We again include caps and floors on the adjustment factors to limit the fluctuations. In this section, the lower bound is set to 0.97 and the upper bound to 1.07 (i.e., 2% higher than the values used in Section 6.3).
4. <u>Hurdle rate adjustments</u>: The initial hurdle rate is set 2% lower than in Section 6.3: it is now the median long-run portfolio return less 2%. The no-action corridor is adjusted accordingly: $h_{\min}$ is set to be 2% lower than this new initial hurdle rate, and $h_{\max}$ is 2% higher than the new initial hurdle rate.

Table 10.

REWARD, RISK, HYBRID, AND SHORTFALL MEASURES FOR DELAYED RECOGNITION OF GAINS AND LOSSES WITH INFLATIONARY INCREASES BUILT INTO THE DESIGN.

		Group		
Measures	**No smoothing**	**Staggered adjustments**	**Adjustment corridor**	**Hurdle rate adjustments**
Expected average benefits (in $000s)				
Go-go years	80.4	75.8	78.6	91.5
Slow-go years	117.0	110.3	112.1	127.0
No-go years	169.5	160.9	162.9	175.3
Expected standard deviation of adjustments				
Go-go years	9.5%	3.2%	3.9%	4.6%
Slow-go years	9.3%	3.7%	3.7%	5.7%
No-go years	9.2%	3.7%	3.7%	5.7%
Expected standard deviation of benefits (in $000s)				
Go-go years	12.2	8.5	8.6	10.5
Slow-go years	22.0	18.7	17.4	23.4
No-go years	30.7	25.7	24.2	31.4
Expected shortfall probability of adjustments, $C_k = 1.00$				
Go-go years	36%	29%	37%	15%
Slow-go years	39%	31%	41%	20%
No-go years	41%	35%	46%	25%
Expected shortfall probability of adjustments, $C_k = 0.95$				
Go-go years	19%	3%	0%	8%
Slow-go years	20%	6%	0%	12%
No-go years	21%	7%	0%	15%
Expected shortfall probability of benefits, $C_k = B_k(0)$				
Go-go years	27%	30%	23%	23%
Slow-go years	15%	15%	11%	25%
No-go years	13%	13%	10%	28%
Expected shortfall probability of benefits, $C_k = 0.75\ B_k(0)$				
Go-go years	3%	1%	0%	4%
Slow-go years	4%	4%	2%	12%
No-go years	5%	5%	4%	18%
Minimum benefit at risk (in $000s)				
5-year horizon	21.1	14.0	9.7	23.8
10-year horizon	20.7	15.1	10.9	26.8
Average benefit at risk (in $000s)				
20-year horizon	29.0	23.7	22.9	34.9
30-year horizon	37.3	32.3	30.9	46.4
40-year horizon	46.1	40.9	39.0	57.3
Certainty equivalent consumption (in $000s)				
20-year horizon	75.3	74.9	79.1	81.2
30-year horizon	76.1	76.1	81.6	72.3
40-year horizon	76.4	76.6	82.9	62.6
50-year horizon	76.2	76.6	83.7	51.8
Relative shortfall value at risk				
5-year horizon	0.0%	7.9%	6.9%	0.0%
10-year horizon	0.0%	7.2%	8.6%	0.0%
20-year horizon	0.0%	7.9%	14.4%	0.0%

Notes: This table reports the various measures introduced in Section 3 and in Section 6 for both open pools and group-based adjustments. We consider pools of 1,000 members at inception in this table. "Group" stands for group-based adjustments.

Figure 19 reports the funnel of doubt for annual benefits of a 65-year-old member at inception when applying the four methods described above. Overall, the average benefit increases as expected: the rate of increase of the average is close to 2% across all four cases. The ordering of the averages is similar to the case investigated in Section 6.3. The hurdle rate adjustment method leads to the smallest average benefits at first but yields larger average benefits than the other methods after about 10–15 years. The base case tends to provide averages that are higher than the staggered adjustment and adjustment corridor methods. The averages shown in Figure 19 are also consistent with the results of Table 10.

The uncertainty around the benefits is similar to that obtained in Section 6.3, when the hurdle rate was set to the long-run median portfolio return: in the short term, the three methods that allow for delayed recognition of gains and losses lead to less uncertainty. Indeed, the funnels of doubt for the staggered adjustment, adjustment corridor, and hurdle rate adjustment methods are narrower than that obtained for the base case for the first 10–15 years. The mBaR values are also compatible with this result, as this measure is mainly capturing the short-term performance of the pool.

This behaviour tends to change after the 10-year mark: the hurdle rate adjustment method yields the most benefit uncertainty, followed by the no-smoothing case, the staggered adjustment method, and the adjustment corridor procedure. The standard deviations of benefits in Table 10 corroborate this story. Shortfall probabilities on benefits are similar across the different methods, except for the hurdle rate adjustment method, which yields values that are about two to four times larger than those of the other cases.

The standard deviations of adjustments tell a somewhat different tale: not smoothing the adjustments leads to the most uncertainty. This is followed by the hurdle rate adjustment method. The staggered adjustment and adjustment corridor methods yield somewhat similar standard deviations of adjustments, albeit they are slightly lower for the staggered adjustment method. This ordering is also similar to that observed with the shortfall probability of adjustments with a threshold of 0.95: the adjustment corridor method is best, followed by the staggered adjustment method, the hurdle rate adjustment method, and finally, the base case.

Long-term measures like the average BaR and the CEC confirm that the long-term uncertainty of the hurdle rate adjustment method negatively impacts pool members: aBaRs are higher and CECs lower across all horizons for this case. The staggered adjustment and adjustment corridor methods yield lower aBaR measures.

For the CECs, the adjustment corridor offers the highest measure; this is followed by the staggered adjustment method and the base case, which yield similar results.

The stable benefit streams obtained with the staggered adjustment and adjustment corridor methods come at a cost, however: the relative shortfall value at risk departs from zero in both these cases, which is similar to Section 6.3. For a horizon of 20 years, the adjustment corridor method has a shortfall value at risk of more than 14% in one scenario out of 20. The value at risk is 8% for the staggered adjustment method.

In summary, this case, which allows for inflation-indexed benefits, produces very similar results, qualitatively speaking, to those presented in Section 6.3. In other words, reducing the initial hurdle rate, which creates likely but not guaranteed increases, does not impact the results reported above.

6.5 COMMENTS ON FAIRNESS AND ABILITY TO ATTRACT NEW MEMBERS

Delaying the recognition of gains and losses in a lifetime pension pool means that prior years' experience may affect future benefit adjustments. While this may be advantageous for pool members in reducing the year-to-year volatility of benefits and the likelihood of extreme outcomes, it can also introduce inequities and impact the pool's capacity to attract new members. It is, therefore, important to ensure that the delayed recognition rules are applied fairly to new entrants.

In fact, the question of fairness in a lifetime pension pool is broader than just delayed recognition, and fairness between existing members and new entrants is only one of its components. It also includes fairness between different age cohorts in the pool, which was the motivating idea behind the cohort-based mortality experience adjustments used by Fullmer and Sabin (2019a). There is also the question of fairness within cohorts. One aspect of this is heterogeneity in the membership with respect to future mortality expectations (e.g., on account of sex or health status); another is the potentially disproportionate impact of an individual's mortality experience on the entire pool (e.g., on account of the size of the initial assets brought in). We do not address these potential issues in this report, focusing only on fairness in the context of delayed recognition of gains and losses.

In relation to new entrants, fairness means that new members' benefits are not unduly affected by experience that unfolded before they joined the pool. For example, with staggered adjustments, if there were past losses that the pool operator knows will be flowing through to the benefits gradually over the next five years, it would not be fair to calculate new members' initial benefits by pricing them as a level stream of payments, unless the new entrants' benefit adjustment factors were calculated separately from the rest of the pool, reflecting only the experience gains and losses that occur after they join. As an alternative, the planned future benefit cuts could be reflected in the pricing at entry, leading to a slightly higher initial benefit, which would then be reduced in line with everyone else's benefits in future years.

The former approach interprets the fairness criterion literally and isolates the benefit adjustments of new entrants. This may not be administratively convenient. Under the latter approach, new entrants would be compensated when they enter the pool for future benefit cuts tied to prior negative experience. Conversely, they would be charged for anticipated future benefit increases tied to prior positive experience.

When using the staggered adjustment method, both approaches are reasonable. In either case, the chosen approach should be communicated clearly to potential new members. When using the adjustment corridor method, unrecognized past experience is built up when the adjustment factor hits the edges of the corridor. This buildup affects the likelihood of future benefit increases (when at the top of the range) and benefit cuts (when at the bottom of the range). Again, this should be taken into account either by applying separate adjustment factors to new entrants or by reflecting it in the pricing of new members' initial benefits. Finally, when using the hurdle rate adjustment method, the hurdle rate itself stores past experience: a low hurdle rate stores past gains, reducing the likelihood of future benefit cuts and magnifying future gains. A high hurdle rate does the opposite. Accounting for this in pricing the initial benefits of new entrants is very simple: one needs only to apply the then-current hurdle rate. When the hurdle rate is low, this will mean a lower initial benefit, effectively charging new members for the increased capacity of the pool to withstand negative experience. When the hurdle rate is high, the initial benefit will be higher, compensating new members for the increased likelihood of future benefit cuts due to negative past experience.

Fair pricing of the initial benefit helps maintain the attractiveness of the pool to new entrants, even when there are unrecognized past losses. However, unrecognized past losses can increase shortfall and bankruptcy risk, which may still deter future members.

The issue of fairness also arises in connection with decedents, not just new entrants. For example, when a pool member dies while there are unrecognized losses, they have effectively borrowed from the pool, yet there is no practical way to recover the outstanding loan amount. This means the mortality gains accruing to surviving members will be smaller than would otherwise be the case (without delayed recognition). In the opposite scenario, deceased members do not receive the full benefit of past gains and leave behind more assets. Although the pool could pay out the excess as a death benefit, this would introduce asymmetry, since outstanding losses could not be reasonably recovered. Lifetime pension pools with delayed recognition of gains and losses should assess whether this impact is material and modify their recognition technique appropriately.

Finally, we note that transparency is very important for the operation of lifetime pension pools: it builds trust between the pool operator and the members, and it simplifies communication. The delayed recognition techniques introduced here differ in their level of transparency, with the staggered adjustment method being the most transparent and the hurdle rate adjustment method being less so. A lack of transparency may fuel suspicions of unfair treatment (as it did for with-profit products in the UK) and may curtail the ability of the pool to attract new members.

Section 7: Concluding Remarks and Further Developments

Lifetime pension pools offer a flexible alternative to traditional guaranteed pension arrangements, enabling retirees to convert a lump sum into a lifelong income that varies based on investment and mortality experience. However, the proliferation of these pools is hindered by three challenges: regulatory issues, the lack of efficient ways to communicate risk to members, and the absence of a consensus on optimal pool design elements.

This report focuses on the third challenge—pool design—exploring the trade-offs associated with three different design elements: open and closed pools, hurdle rate policy, and delayed recognition of gains and losses. The report finds that most design elements, such as the following, involve trade-offs between risk and reward, highlighting the need for careful consideration by pool operators when selecting their assumptions.

1. Closed and open pools: Open pools typically pose less risk than closed ones, particularly when the latter pools consist mostly of older members.
2. Hurdle rate policy: Compared to a fixed hurdle rate, using a variable hurdle rate decreases the standard deviation of adjustments and benefits. It might also improve fairness among generations. Nonetheless, a variable hurdle rate policy might be a challenge from a communication perspective for pool operators. Therefore, pool operators and designers should exercise caution when selecting their hurdle rate assumptions.
3. Delayed recognition of gains and losses: Delayed recognition of gains and losses can significantly decrease the risk associated with modifying benefits. However, this approach often increases the risk of shortfall and bankruptcy, which could impact fairness and the pool's ability to attract new members.

We only considered three methods for delaying the recognition of gains and losses in this report; there might be many others in actuarial practice and in the academic literature (see, e.g., Guillén et al., 2006). Moreover, our implementation of these procedures was not optimal in any way—they were selected based on common sense. More research would be needed to find optimal ways to set gain and loss delay mechanisms. Given the conclusions of Section 6, we believe it could be worthwhile to explore these mechanisms in greater depth, as they can potentially help with the proliferation of lifetime pension pools. One should be cautious, however, to ensure that the potential gains in terms of benefit smoothing are not creating significant fairness issues, as explained in Section 6.5.

The three elements of pool design investigated in this report are relevant; we acknowledge, nonetheless, that this is only a starting point and that many other elements could have been considered (e.g., single- versus multi-cohort pools, patterns of expected benefit payments, refund or death benefit options, and investment policies). We leave these exciting questions for future research.

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Appendix A: Economic Scenario Generator

Over the last four decades, many different frameworks have been proposed to model economic and financial variables relevant to actuaries. These frameworks—called *economic scenario generators* or *ESGs*—are comprehensive models that allow actuaries and risk managers to understand the long-term uncertainty underlying financial market values and economic variables. The primary end-users of these frameworks are pension, life insurance, and banking practitioners, who use them for various purposes (see, e.g., Pedersen et al., 2016, for a general introduction to economic scenario generators).

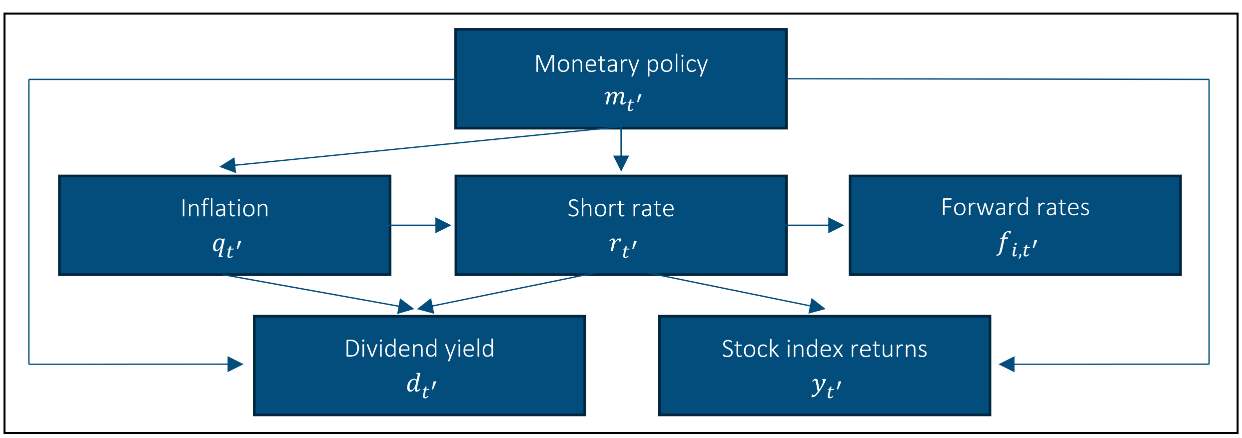
Wilkie proposed the first cascade framework in a pioneering 1986 article. His framework, which relies on the Box–Jenkins approach, is based on four models: an inflation model, a long interest rate model, a dividend yield model, and a stock index return model. These four models are interconnected. In a follow-up article, Wilkie (1995) generalized the model and allowed for an earnings index, short-term interest rates, and property prices.

Modern technological and methodological advances have paved the way for more advanced models. Specifically, Ahlgrim et al. (2005) proposed a model similar to Wilkie's but allowed for regime-switching dynamics for stock index returns to capture bull and bear markets. More recently, Bégin (2021) introduced a new cascade-type ESG based on the monetary policy, which is modelled via observable regime-switching dynamics. The model also considers the changing nature of the volatility via generalized autoregressive conditional heteroskedasticity (GARCH) models. In the present report, we rely on a simplified version of this model. Specifically:

1. We use a homoskedastic autoregressive model for price inflation and the dividend yield with (monetary) regime-dependent, long-run levels, as supported by the conclusions of Bégin (2023) for the Canadian economy.
2. We rely on more sophisticated models for the short rate and the stock index returns. Specifically, we use GARCH models to capture heteroskedasticity and a regime-switching component based on the monetary policy to capture changes in the level.
3. The risk-free term structure is constructed based on observable factors: the level (proxied by the short rate), the slope, and the curvature of the term structure. The slope and the curvature are modelled by a two-dimensional autoregressive model.

Figure 20 shows the general structure of the economic scenario generator used in this report; for more information about the ESG, see Bégin (2021, 2023). Note that each time series is observed on a monthly basis (and months are denoted by t' in this report).

Figure 20.
CASCADE STRUCTURE OF THE ECONOMIC SCENARIO GENERATOR.



A.1 MONETARY POLICY REGIMES

The primary level of our proposed ESG models the monetary policy. We suggest a regime-switching dynamics to capture the changing nature of the policy, in the spirit of Renne (2017). Let $m_{t'}$ be a discrete-time observable Markov chain with three states: tightening or *upward* (u), status quo (s), and accommodating or *downward* (d). The transition probabilities of this Markov chain are given as follows:

$$\Pi = \begin{bmatrix} p_{\mathrm{uu}} & p_{\mathrm{us}} & 0 \\ p_{\mathrm{su}} & p_{\mathrm{ss}} & p_{\mathrm{sd}} \\ 0 & p_{\mathrm{ds}} & p_{\mathrm{dd}} \end{bmatrix} = \begin{bmatrix} p_{\mathrm{uu}} & 1 - p_{\mathrm{uu}} & 0 \\ p_{\mathrm{su}} & p_{\mathrm{ss}} & p_{\mathrm{sd}} \\ 0 & 1 - p_{\mathrm{dd}} & p_{\mathrm{dd}} \end{bmatrix}$$

because $p_{\mathrm{uu}} + p_{\mathrm{us}} = 1$, $p_{\mathrm{su}} + p_{\mathrm{ss}} + p_{\mathrm{sd}} = 1$, and $p_{\mathrm{ds}} + p_{\mathrm{dd}} = 1$.

We infer the states of this Markov chain from a reference rate $R_{t'}$—fixed by the central bank—following

$$m_{t'} = \begin{cases} \mathrm{u} & \text{if } \exists\, u \in [t'-3, t'] \text{ and } v \in [t', t'+3] \text{ such that } R_{t'} - R_u > 0 \text{ and } R_v - R_{t'} > 0 \\ \mathrm{d} & \text{if } \exists\, u \in [t'-3, t'] \text{ and } v \in [t', t'+3] \text{ such that } R_{t'} - R_u > 0 \text{ and } R_v - R_{t'} > 0 \\ \mathrm{s} & \text{otherwise} \end{cases}.$$

In lay terms, this state extraction method considers an upward regime at time t' whenever we see an increase in the reference rate. Similarly, we consider a downward regime at time t' whenever we observe a decrease in the reference rate.

A.2 INFLATION MODEL

Let $\mathrm{CPI}_{t'}$ be the level of the consumer price index (CPI) at time t'. For most economies, the CPI time series contain a unit root. Hence, instead of modelling the level itself, it is common to model the rate of change of the CPI. For this reason, we use AR(1) dynamics to model inflation rate as used in other frameworks, such as Wilkie (1986, 1995) and Ahlgrim et al. (2005), with one main distinction: the long-run level of the inflation is now regime-dependent (i.e., $\mu_{q,\mathrm{u}}$ in the upward regime, $\mu_{q,\mathrm{s}}$ in the status quo regime, and $\mu_{q,\mathrm{d}}$ in the downward regime). The time-t' inflation rate is therefore given as follows:

$$\begin{aligned} q_{t'} &= \log\left(\frac{\mathrm{CPI}_{t'}}{\mathrm{CPI}_{t'-1}}\right) \\ &= \mu_{q,m_{t'}} + a_q\left(q_{t'-1} - \mu_{q,m_{t'}}\right) + \sigma_q \varepsilon_{q,t'}, \end{aligned}$$

where the innovation $\varepsilon_{q,t'}$ is a standardized normal, or $\varepsilon_{q,t'} \sim \mathcal{N}(0,1)$. Parameters $\mu_{q,\mathrm{u}}$, $\mu_{q,\mathrm{s}}$, and $\mu_{q,\mathrm{d}}$ are the regime-dependent long-run levels of the inflation rate, α_q is the autoregressive parameter, and σ_q^2 is the inflation variance parameter.

A.3 SHORT RATE

Our framework's risk-free interest rate is comprised of two components: a short rate model and a term structure component constructed on top of it. We introduce each component separately, and this subsection focuses on the former.

The literature on ESG shows divisions on the issue of negative rates: Wilkie (1986, 1995) uses a logarithmic transformation to ensure that real interest rates are positive, whereas Ahlgrim et al. (2005) allow for negative rates. Our model embeds both approaches. Specifically, we rely on a transformation reminiscent of Engle et al. (2017) by incorporating a piecewise function that deals differently with high and low rates: we use a linear transform for higher rates and a logarithmic transform for lower rates. The transformed (continuously compounded) short rate is given by

$$\tilde{r}_{t'} = \begin{cases} r_{t'} & \text{if } r_{t'} > 0.005 \\ 0.005 + 0.005(\log(r_{t'}) - \log(0.005)) & \text{if } r_{t'} \leq 0.005 \end{cases}.$$

Similar to the inflation model introduced above, the transformed short rate is modelled by an AR(1) model with a regime-dependent mean level, to which we add conditional heteroskedasticity:

$$\tilde{r}_{t'} = \mu_{r,m_{t'}} + a_r\left(\tilde{r}_{t'-1} - \mu_{r,m_{t'}}\right) + \sigma_{r,t'}\varepsilon_{r,t'}, \qquad \varepsilon_{r,t'} \sim \mathcal{N}(0,1),$$
$$\sigma^2_{r,t'+1} = \sigma^2_r + \alpha_r\left(\left(\sigma_{r,t'}\varepsilon_{r,t'} - \sigma_{r,t'}\gamma_r\right)^2 - \sigma^2_r(1+\gamma^2_r)\right) + \beta_r\left(\sigma^2_{r,t} - \sigma^2_r\right),$$

where $\mu_{r,\mathrm{u}}$, $\mu_{r,\mathrm{s}}$, and $\mu_{r,\mathrm{d}}$ are the regime-dependent, long-run levels of the short rate. Moreover, σ^2_r is the long-run level of the variance, and α_r, β_r, and γ_r are the GARCH reaction, persistence, and asymmetry parameters, respectively.

To capture the potential relationship between inflation and the short-rate innovations, we assume non-zero correlation between $\varepsilon_{q,t'}$ and $\varepsilon_{r,t'}$; that is, $\mathrm{Corr}\left(\varepsilon_{r,t'}, \varepsilon_{r,t'}\right) = \rho_{q,r}$.

A.4 TERM STRUCTURE

To capture the rest of the term structure, we model (continuously compounded) forward rates constructed from yields. Let $f_{i,t'}$ be the forward rate observed at time t' for a contract starting at $t' + \tau_{i-1}$ and ending at the next available maturity, $t' + \tau_i$ such that

$$f_{i,t'} = \frac{1}{\tau_i - \tau_{i-1}}\left(\tau_i r_{\tau_i,t'} - \tau_{i-1} r_{\tau_{i-1},t'}\right), \qquad i \in \{1,2,\dots,n\},$$

where $r_{\tau_i,t}$ is the zero-coupon bond yield for tenor τ_i, and the shortest maturity available is assumed to be the short rate (i.e., $r_{\tau_0,t} = r_t$).

To prevent the forward rates from becoming negative, we apply a similar transformation to that used for the short rate; that is,

$$\tilde{f}_{i,t'} = \begin{cases} f_{i,t'} & \text{if } f_{i,t'} > 0.005 \\ 0.005 + 0.005\left(\log(f_{i,t'}) - \log(0.005)\right) & \text{if } f_{i,t'} \leq 0.005 \end{cases}.$$

Specifically, the difference between the transformed rates and the transformed short rate is then modelled using two observable factors—the slope and the curvature.[43,44] The term structure is thus obtained by assuming that the forward rates are generated by the following equation:

$$\tilde{f}_{i,t'} = \tilde{r}_{t'} + \mu_{f_i} + a_{f_i,1}F_{1,t'} + a_{f_i,2}F_{2,t'} + \sigma_{f_i}\varepsilon_{f_i,t'}, \qquad \varepsilon_{f_i,t'} \sim \mathcal{N}(0,1),$$

where, again, $i \in \{1,2,\dots,n\}$.

As commonly done in the literature, the factors—the slope and the curvature—are calculated in the following way:

[43] Note that the level will be indirectly accounted for by having the short rate as a part of our (transformed) forward rate model.

[44] Litterman and Scheinkman (1991) show that 99% of the yield curve's total variation can be explained by three fundamental shifts: a level component, a slope component, and a curvature component.

$$F_{1,t'} = \tilde{f}_{n,t'} - \tilde{f}_{1,t'},$$
$$F_{2,t'} = \tilde{f}_{1,t'} + \tilde{f}_{n,t'} - 2\tilde{f}_{3,t'},$$

and are modelled by a two-dimensional autoregressive model

$$F_{j,t'} = \mu_{F_j} + a_{F_j}\left(F_{j,t'-1} - \mu_{F_j}\right) + \sigma_{F_j}\varepsilon_{F_j,t'}, \qquad \varepsilon_{F_j,t'} \sim \mathcal{N}(0,1),$$

where $j \in \{1,2\}$. Rates that are not readily available in our model (i.e., rates with tenors different than τ_1 to τ_n) are interpolated in the term structure.

A.5 DIVIDEND YIELD

Similar to the ESGs of Wilkie (1986, 1995) and Ahlgrim et al. (2005), the logarithm of the dividend yield relies on AR(1) dynamics:

$$\log(d_{t'}) = \log\left(\mu_{d,m_{t'}}\right) + a_d\left(\log(d_{t'-1}) - \log\left(\mu_{d,m_{t'}}\right)\right) + \sigma_d\varepsilon_{d,t'}, \qquad \varepsilon_{d,t'} \sim \mathcal{N}(0,1),$$

where the interpretation of parameters is similar to their inflation and short rate counterparts. Also, identically to the relationship between inflation and the short rate, we assume a non-nil correlation between $\varepsilon_{q,t'}$ and $\varepsilon_{d,t'}$, and between $\varepsilon_{r,t'}$ and $\varepsilon_{d,t'}$.

A.6 STOCK INDEX RETURNS

The stock index returns are given by a process reminiscent of that used by Hardy (2001). Instead of considering latent regimes, however, the model uses observable monetary regimes, which capture the changing nature of the average return. In addition, we add a GARCH structure to capture the changing nature of volatility over time. Assuming that $S_{t'}$ is the time-t' price of the index and $y_{t'}$ is the time-t' return over one month, we have the following dynamics:

$$\begin{aligned} y_{t'} &= \log\left(\frac{S_{t'}}{S_{t'-1}}\right) \\ &= \frac{r_{t'}}{12} + \mu_{y,m_{t'}} + \sigma_{y,t'}\,\varepsilon_{y,t'}, \qquad \varepsilon_{y,t'} \sim \mathcal{N}(0,1), \\ \sigma^2_{y,t'} &= \sigma^2_y + \alpha_y\left(\left(\sigma_{y,t'}\varepsilon_{y,t'} - \sigma_{y,t'}\gamma_y\right)^2 - \sigma^2_y\left(1+\gamma^2_y\right)\right) + \beta_y\left(\sigma^2_{y,t'} - \sigma^2_y\right), \end{aligned}$$

where the interpretation of parameters is similar to that of other models.

A.7 DATA AND RESULTS

In this report, we use monthly Canadian economic and financial data to estimate the parameters of the ESG. Our sample period extends from 1993 to 2022. Details about the datasets used in this report are given below:

1. The model requires a reference interest rate to construct monetary regimes. We select the Bank of Canada's target overnight rate; we find the target overnight rate on the Bloomberg terminal.
2. The monthly inflation rate series are constructed from non-seasonally adjusted CPIs (all items). Specifically, the CPI level data are converted into monthly rates by taking the (logged) returns of the index. The Canadian CPI is extracted from Statistics Canada.
3. The monthly interest rate term structures are based on the series extracted from the Bank of Canada's website. The three-month rate is used as a proxy for the short rate. We also use one-, two-, three-, five-, seven-, ten-, and 20-year interest rates to estimate the term structure model.

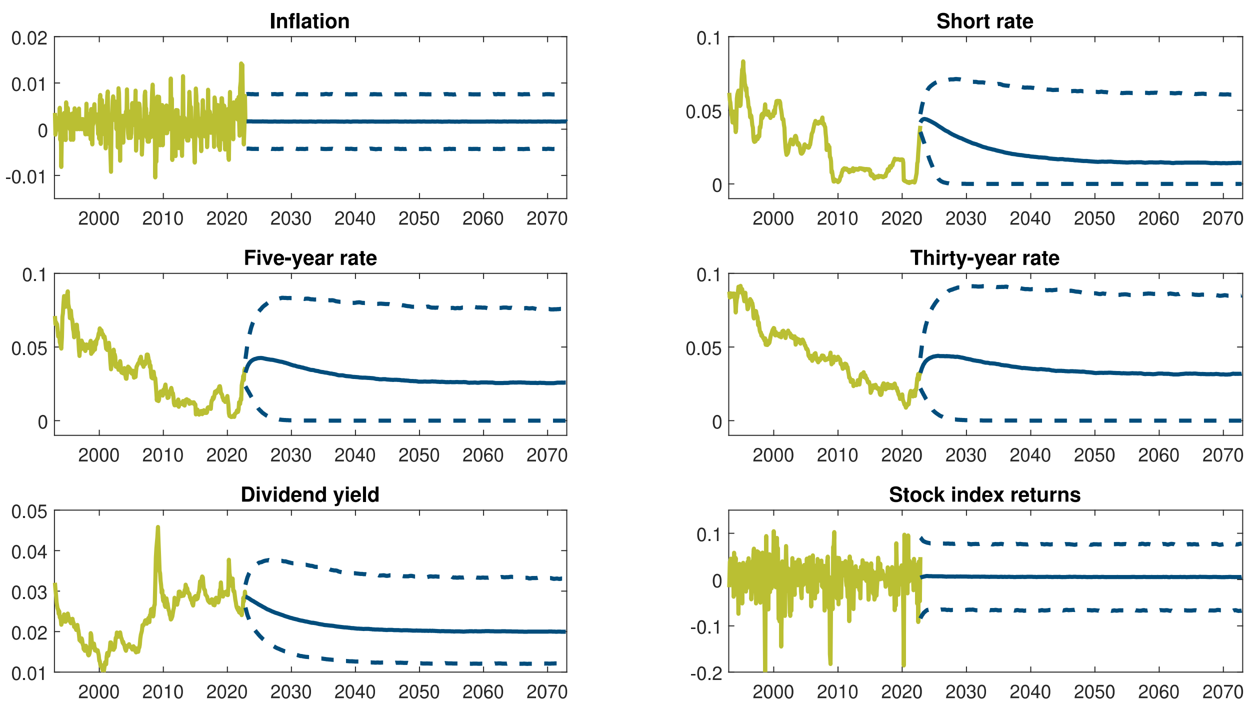
4. The stock index dividend yield series are proxied by the dividends paid out on the stocks of the index—the S&P/TSX Composite index. They are extracted from the Wharton Research Data Services (WRDS) and constructed by taking the sum of the gross dividend payments over the previous 12 months and dividing it by the value of the index at the beginning of this period.
5. The stock index returns are constructed from the index level and also obtained from WRDS, as explained above for the dividend yield. Similar to our process for inflation, we build our results out of monthly index values.

The parameters are obtained by the maximum likelihood estimation method.

Figure 21 reports monthly funnels of doubt taken at the end of our sample for inflation, the short rate, the five- and 30-year interest rate yields, the dividend yield, and the stock index returns. At the end of 2022, interest rates were rising as the Bank of Canada announced a series of rate hikes over the last year of our sample. The stock market volatility was also at a higher level than its long-run level, meaning that returns in 2023 are expected to be more volatile. It returns to its long-run level after a few years, nonetheless (see bottom right panel of Figure 21).

Figure 21.

MONTHLY FUNNELS OF DOUBT OF RELEVANT ECONOMIC AND FINANCIAL VARIABLES.



Notes: This figure shows the past series (in green) along with the funnels of doubt over the next 50 years (in blue). The figure reports monthly values for inflation, the short rate, the five-year interest rate, the 30-year interest rate, the dividend yield on the S&P/TSX Composite index, and the S&P/TSX Composite index returns. Solid lines represent the median and dashed lines the 90% confidence interval.

Monthly data and series are then combined to obtain annual quantities needed to simulate the lifetime pension pool operation over time.

Appendix B: Mortality Modelling

Interestingly, most of the literature on lifetime pension pools focuses on the effect of (idiosyncratic) mortality on the benefit stream (see, e.g., Piggott et al., 2005, Qiao and Sherris, 2013, Olivieri and Pittaco, 2020), giving less regard to the impact of the realized rate of return on the asset portfolio. In this report, we account for both dimensions of the adjustments: investment and mortality.

B.1 MORTALITY TABLE

The present report uses a deterministic generational mortality table to model deaths. We rely on the CPM 2014 table, with CPM Improvement Scale B provided by the Canadian Institute of Actuaries (see CIA, 2014, for more details on the table's construction).

The CPM 2014 table is a table for base Canadian pensioners' mortality as of 2014 that uses the combined experience of the Canadian public and private sector plans from 1999 to 2008. We add improvements to this base table that are captured by an improvement scale; the latter scale considers mortality improvement trends in the future based on the observed trends in the Canadian mortality experience since 1967. The CPM Improvement Scale B used in this report provides improvement rates by age that decrease linearly from 2012 to 2030 and ultimate rates for all years after 2030. The post-2014 expected mortality rates are therefore given as follows:

$$q_{x+t}^{[s]} = q_{x+t} \prod_{u=1}^{t} \left(1 - i_{x+t,u}^{[s]}\right),$$

where $q_{x+t}^{[s]}$ is the one-year death probability at time t for a member aged x at inception (i.e., time 0) as per the CPM 2014 table with improvements. Moreover, q_{x+t} is the one-year death probability for a member aged $x+t$ as per the base table, and $i_{x+t,u}^{[s]}$ is the improvement rate in mortality for members aged $x+t$ at time u given by the Improvement Scale B. Note that the superscript—in brackets—refers again to the assumptions used to compute the annuity due prices; for instance, $[2014]$ means that we use the mortality table per the 2014 estimation by the Canadian Institute of Actuaries.

B.2 ANNUITY-DUE PRICE CALCULATION

The price of an annuity due for a member aged x at inception is given by

$$\ddot{a}_{x,t}^{[s]} = \sum_{u=0}^{\infty} {}_{u}p_{x+t}^{[s]} \exp\left(-uh^{[s]}\right).$$

If the term structure is not flat—as in Section 5—the hurdle rate applicable to a specific cash flow depends on both the time of valuation and the time at which the cash flow is paid. In this case, we replace the discount factor at time u, $\exp\left(-uh^{[s]}\right)$, with the discount factor $\exp\left(-uh_u^{[s]}\right)$, using the correct rate matching the maturity of the cash flow.

B.3 IDIOSYNCRATIC MORTALITY MODELLING

Once we know the various death probabilities, we can model the idiosyncratic mortality within a given pool. To account for this dimension, we need to keep track of the members in $\mathcal{L}_t$ for $t \in \{0,1,\ldots\}$. Assume a pool of N members at inception so that $\mathcal{L}_0 = \{1,2,\ldots,N\}$. Using a recursion, we can then define $\mathcal{L}_t$ from $\mathcal{L}_{t-1}$ by keeping track of the decedents and the survivors. For instance, if the k^{th} member is alive at time $t-1$ (i.e., $k \in \mathcal{L}_{t-1}$), then there are two possibilities for their status at time t:

1. They will survive until at least time t with probability $p_{x_k+t-1}^{[s]} = 1 - q_{x_k+t-1}^{[s]}$—in this case, $k \in \mathcal{L}_t$.

2. They will die between $t-1$ and t with probability $q_{x_k+t-1}^{[s]}$—in this case, $k \notin \mathcal{L}_t$.

This behaviour is reminiscent of a Bernoulli random variable. Indeed, if $k \in \mathcal{L}_{t-1}$, then

$$\begin{cases} k \in \mathcal{L}_t & \text{with probability } p_{x_k+t-1}^{[s]} \\ k \notin \mathcal{L}_t & \text{with probability } q_{x_k+t-1}^{[s]} \end{cases},$$

where success (first outcome) is survival and failure (second outcome) is death.

B.4 OTHER MORTALITY-RELATED RISKS

For simplicity, we only considered idiosyncratic mortality risk in this report: we have not considered the possibility of the pool operator using the wrong base mortality table or of misestimating the magnitude of mortality improvements. Both of these would give rise to additional mortality experience adjustments, either retrospectively (in response to emerging experience varying more from expected) or prospectively (in response to adjustments to the mortality assumptions used in pricing the benefits). Including these additional features would most likely increase the various risk measures calculated in this report. Aside from the challenge of allocating these other mortality risks equitably between cohorts, we do not anticipate that this change will have a material qualitative impact on our main results. We leave the interesting question of assessing the importance of these various mortality risks in the context of lifetime pension pools for future research.

References

Ahlgrim, K. C., D'Arcy, S. P., and Gorvett, R. W. (2005). Modeling financial scenarios: A framework for the actuarial profession. *Proceedings of the Casualty Actuarial Society*, *92*(177): 177–238.

Association of Canadian Pension Management. (2017). *Decumulation, the next critical frontier: Improvements for defined contribution and capital accumulation plans*.

Australian Government Actuary. (2014). *Towards more efficient retirement income products*.

Balter, A. G., and Werker, B. J. (2020). The effect of the assumed interest rate and smoothing on variable annuities. *ASTIN Bulletin*, *50*(1), 131–154.

Balter, A. G., Kallestrup-Lamb, M., and Rangvid, J. (2020). Variability in pension products: A comparison study between The Netherlands and Denmark. *Annals of Actuarial Science*, *14*(2), 338–357.

Barsky, R. B., Juster, F. T., Kimball, M. S., and Shapiro, M. D. (1997). Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement study. *Quarterly Journal of Economics*, *112*(2), 537–579.

Bégin, J.-F. (2021). On complex economic scenario generators: Is less more? *ASTIN Bulletin*, *51*(3), 779–812.

Bégin, J.-F. (2023). Ensemble economic scenario generators: Unity makes strength. *North American Actuarial Journal*, *27*(3), 444–471.

Bégin, J.-F. and Sanders, B. (2023). *Benefit at risk in lifetime pension pools*. Society of Actuaries and Canadian Institute of Actuaries.

Bernhardt, T., and Donnelly, C. (2019). Modern tontine with bequest: Innovation in pooled annuity products. *Insurance: Mathematics and Economics*, *86*, 168–188.

Bernhardt, T., and Donnelly, C. (2021). Quantifying the trade-off between income stability and the number of members in a pooled annuity fund. *ASTIN Bulletin*, *51*(1), 101–130.

Blanchett, D. (2014). Exploring the retirement consumption puzzle. *Journal of Financial Planning*, *27*(5), 34–42.

Boyle, P., Hardy, M., Mackay, A., and Saunders, D. (2015). *Variable payout annuities*. Society of Actuaries.

Canadian Institute of Actuaries. (2014). *Canadian pensioners' mortality*.

Chen, A., and Rach, M. (2019). Options on tontines: An innovative way of combining tontines and annuities. *Insurance: Mathematics and Economics*, *89*, 182–192.

Chen, A., and Rach, M. (2023). The tontine puzzle. *Working Paper*.

Chen, A., Hieber, P., and Klein, J. K. (2019). Tonuity: A novel individual-oriented retirement plan. *ASTIN Bulletin*, *49*(1), 5–30.

Chen, A., Rach, M., and Sehner, T. (2020). On the optimal combination of annuities and tontines. *ASTIN Bulletin*, *50*(1), 95–129.

Chen, A., Qian, L., and Yang, Z. (2021). Tontines with mixed cohorts. *Scandinavian Actuarial Journal*, *2021*(5), 437–455.

Chen, A., Nguyen, T., and Sehner, T. (2022). Unit-linked tontine: Utility-based design, pricing and performance. *Risks*, *10*(4), 78.

College Retirement Equities Fund. (2022a). *Prospectus*. Retrieved June 27, 2022, from https://connect.rightprospectus.com/TIAA/TADF/194408803/S?site=VA.

College Retirement Equities Fund. (2022b). *Statement of additional information*. Retrieved June 27, 2022 from http://connect.rightprospectus.com/TIAA/TADF/194408803/S?site=VA&fundclass=RPVA

Davis, R. (2019). *Quantitative evaluation framework for retirement benefit systems*. Society of Actuaries.

Dhaene, J., Goovaerts, M., and Van Weert, K. (2008). Some comments on QIS3. *Zavarovalniski Horizonti*, *3*, 73–87.

Donnelly, C., Guillén, M., and Nielsen, J. P. (2013). Exchanging uncertain mortality for a cost. *Insurance: Mathematics and Economics*, *52*(1), 65–76.

Donnelly, C., Guillén, M., and Nielsen, J. P. (2014). Bringing cost transparency to the life annuity market. *Insurance: Mathematics and Economics*, *56*, 14–27.

Donnelly, C. (2015). Actuarial fairness and solidarity in pooled annuity funds. *ASTIN Bulletin*, *45*(1), 49–74.

Donnelly, C. (2023). Heterogeneity among new entrants to a pooled annuity fund. *Working Paper*.

Eadie, S. A. (2017). *An improved application of the variable annuity*. Global Risk Institute.

Engle, R., Roussellet, G., and Siriwardane, E. (2017). Scenario generation for long run interest rate risk assessment. *Journal of Econometrics*, *201*(2), 333–347.

Forman, J. B., and Sabin, M. J. (2015). Tontine pensions. *University of Pennsylvania Law Review*, *163*(3), 755–831.

Fullmer, R. K. (2019). *Tontines: A practitioner's guide to mortality-pooled investments*. CFA Institute Research Foundation.

Fullmer, R. K., and Sabin, M. J. (2019a). Individual tontine accounts. *Journal of Accounting and Finance*, *19*(8), 2158–3625.

Fullmer, R. K., and Sabin, M. J. (2019b). Tontine bond ladders. *Working Paper*.

Fullmer, R. K., and Forman, J. B. (2022). State-sponsored pensions for private-sector workers. In Mitchell, O. (Ed.), *New Models for Managing Longevity Risk: Public-Private Partnerships* (pp. 171–206). Oxford Academic.

Gerber, H. U. and Pafumi, G. (1998). Utility functions: From risk theory to finance. *North American Actuarial Journal*, *2*(3), 74–91.

Guillén, M., Jørgensen, P. L., and Nielsen, J. P. (2006). Return smoothing mechanisms in life and pension insurance: Path-dependent contingent claims. *Insurance: Mathematics and Economics*, *38*(2), 229–252.

Hanewald, K., Piggott, J., and Sherris, M. (2013). Individual post-retirement longevity risk management under systematic mortality risk. *Insurance: Mathematics and Economics*, *52*(1), 87–97.

Hardy, M. R. (2001). A regime-switching model of long-term stock returns. *North American Actuarial Journal*, *5*(2), 41–53.

Horneff, W. J., Maurer, R. H., Mitchell, O. S., and Stamos, M. Z. (2010). Variable payout annuities and dynamic portfolio choice in retirement. *Journal of Pension Economics & Finance*, *9*(2), 163–183.

Iwry, J. M., Haldeman, C., Gale, W., and John, D. (2020). *Retirement tontines: Using a classical finance mechanism as an alternative source of retirement income*. The Brookings Institution.

Litterman, R., and Scheinkman, J. (1991). Common factors affecting bond returns. *Journal of Fixed Income*, *1*(1), 54–61.

MacDonald, B.-J., Jones, B., Morrison, R. J., Brown, R. L., and Hardy, M. (2013). Research and reality: A literature review on drawing down retirement financial savings. *North American Actuarial Journal*, *17*(3), 181–215.

MacDonald, B.-J., Sanders, B., Strachan, L., and Frazer, M. (2021). *Affordable Lifetime Pension Income for a Better Tomorrow*. Global Risk Institute and National Institute on Ageing.

Milevsky, M. A., and Salisbury, T. S. (2015). Optimal retirement income tontines. *Insurance: Mathematics and Economics*, *64*, 91–105.

Milevsky, M. A., and Salisbury, T. S. (2016). Equitable retirement income tontines: Mixing cohorts without discriminating. *ASTIN Bulletin*, *46*(3), 571–604.

Olivieri, A., and Pitacco, E. (2020). Linking annuity benefits to the longevity experience: Alternative solutions. *Annals of Actuarial Science*, *14*(2), 316–337.

Pedersen, H., Campbell, M. P., Christiansen. S. L., Cox, S. H., Finn, D., Griffin, K., Hooker, N., Lightwood, M., Sonlin, S. M., and Suchar, C. (2016). *Economic scenario generators: A practical guide*. Society of Actuaries.

Piggott, J., Valdez, E. A., and Detzel, B. (2005). The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance*, *72*(3), 497–520.

Pollak, R. A. (1970). Habit formation and dynamic demand functions. *Journal of Political Economy*, *78*(4), 745–763.

Qiao, C., and Sherris, M. (2013). Managing systematic mortality risk with group self-pooling and annuitization schemes. *Journal of Risk and Insurance*, *80*(4), 949–974.

Renne, J. P. (2017). A model of the euro-area yield curve with discrete policy rates. *Studies in Nonlinear Dynamics & Econometrics*, *21*(1), 99–116.

Sabin, M. J. (2010). Fair tontine annuity. *Working Paper*.

Sabin, M. J., and Forman, J. B. (2016). The analytics of a single-period tontine. *Working Paper*.

Sanders, B. (2016). *Analysis of target benefit plan design options*. Society of Actuaries.

Shemtob, M. (2021, September). CRITs: An alternative retirement income approach. *Retirement Section News of the Society of Actuaries*.

Shemtob, M. (2022, September/October). Time to take the plunge. *Contingencies*, 32–36.

Stamos, M. Z. (2008). Optimal consumption and portfolio choice for pooled annuity funds. *Insurance: Mathematics and Economics*, *43*(1), 56–68.

Stevenson, R. (2019). *Actuarial utility and preference functions.* Society of Actuaries.

Valdez, E. A., Piggott, J., and Wang, L. (2006). Demand and adverse selection in a pooled annuity fund. *Insurance: Mathematics and Economics*, *39*(2), 251–266.

Wilkie, A. D. (1986). A stochastic investment model for actuarial use. *Transactions of the Faculty of Actuaries*, *39*, 341–403.

Wilkie, A. D. (1995). More on a stochastic asset model for actuarial use. *British Actuarial Journal*, *1*(5), 777–964.

Winter, P., and Planchet, F. (2022). Modern tontines as a pension solution: A practical overview. *European Actuarial Journal*, *12*(1), 3–32.

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